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ABSTRACT

This document outlines the Mathematics Curriculum Framework Project and provides details on the mathematics strands, standards, and benchmarks in the state of Michigan. Many activities related to the strands, standards, and benchmarks are presented for elementary, middle, and high school levels. A CD-ROM, a list of additional publications, and Internet links are also included. (DDR)

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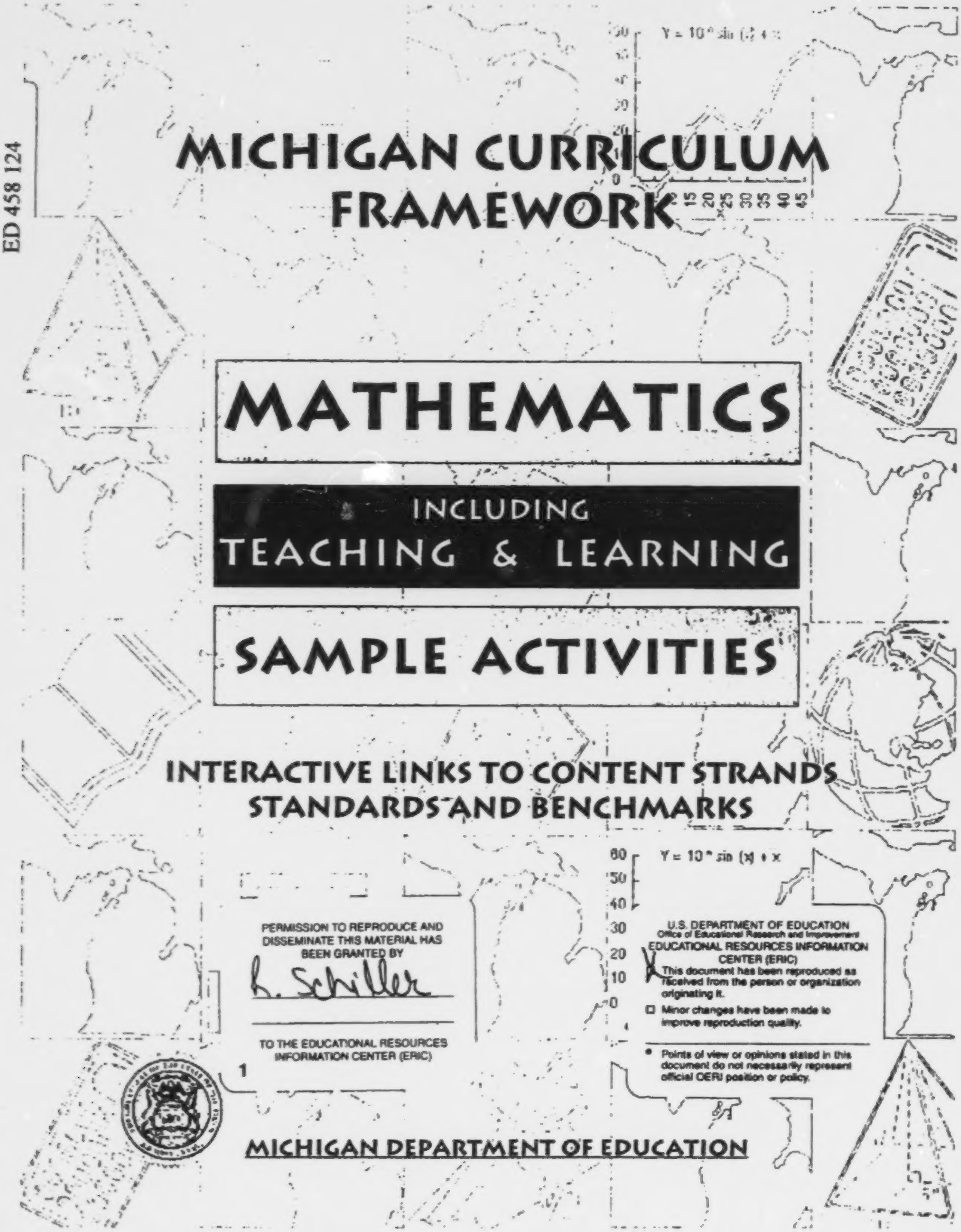

MICHIGAN CURRICULUM FRAMEWORK

MATHEMATICS

INCLUDING TEACHING & LEARNING

SAMPLE ACTIVITIES

INTERACTIVE LINKS TO CONTENT STRANDS STANDARDS AND BENCHMARKS

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MATHEMATICS CURRICULUM FRAMEWORK PROJECT
INTRODUCTION

Vision:

Mathematics is the science of patterns and relationships. It is the language and logic of our technological world. Mathematical power is the ability to explore, to conjecture, to reason logically and to use a variety of mathematical methods effectively to solve problems. The ultimate goal of mathematics education is for all students to develop mathematical power to participate fully as a citizen and worker in our contemporary world.

A mathematically powerful individual should be able to:

- reason mathematically;
- communicate mathematically;
- problem-solve using mathematics; and
- make connections within mathematics and between mathematics and other fields.

EDITOR'S NOTE

In the Mathematics activities section, ✪ and ⊕ indicate content intended for the continued study of mathematics.

Example 1 Example 2

What does the Framework mean to me?

Kathy Grzesiak
Teacher, East Lawn Elementary
Midland Community Schools

"...It's not only just the teacher learning what the words mean, but the teacher being able to translate it into classroom practices..."

Is the Michigan Curriculum Framework a document which must be followed point by point?

Elizabeth Jones, Teacher
Walnut Elementary
Lansing Public Schools

"...the Framework establishes some big ideas... it talks about what we value in the community of mathematics..."

Are Benchmark examples helpful?

Kim Cotter, Director
Curriculum, Special Education and
Professional Development
Portland Community Schools

"...We work together in curriculum teams... and are very interested and concerned in what gets taught and how it gets taught..."

MATHEMATICS CURRICULUM FRAMEWORK PROJECT
MATHEMATICS STRANDS

The Mathematics Framework content has been categorized into the following six strands:

I. **Patterns, Relationships and Functions**
Students recognize similarities among objects and events, generalize patterns and relationships, and use them to describe the physical world, to explain variation, and to make predictions and solve problems.

II. **Geometry and Measurement**
Students use analytical and spatial concepts of shape, size, position, measurement and dimension to understand and interpret the three-dimensional world in which we live.

III. **Data Analysis and Statistics**
Students organize, interpret and transform data into useful knowledge to make predictions and decisions based on data.

IV. **Number Sense and Numeration**
Students quantify and measure objects, estimate mathematical quantities, and represent and communicate ideas in the language of mathematics.

V. **Numerical and Algebraic Operations and Analytical Thinking**
Students represent quantitative situations with numerical and algebraic symbolism and use analytic thinking to solve problems in significant contexts and applications.

VI. **Probability and Discrete Mathematics**
Students deal with uncertainty, make informed decisions based on evidence and expectations, exercise critical judgment about conclusions drawn from data, and apply mathematical models to real-world phenomena.

15 mathematics standards have been written within the six content strands-->

MATHEMATICS CURRICULUM FRAMEWORK PROJECT
MATHEMATICS STANDARDS

The following 15 mathematics standards have been written within the six content strands as follows:

I. Patterns, Relationships and Functions

1. **Patterns** - Students recognize similarities and generalize patterns, use patterns to create models and make predictions, describe the nature of patterns and relationships, and construct representations of mathematical relationships.
2. **Variability and Change** - Students describe the relationships among variables, predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change.

II. Geometry and Measurement

1. **Shape and Shape Relationships** - Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes, identify properties and describe relationships among shapes.
2. **Position** - Students identify locations of objects, identify location relative to other objects, and describe the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object.
3. **Measurement** - Students compare attributes of two objects or of one object with a standard (unit), and analyze situations to determine what measurement(s) should be made and to what level of precision.

III. Data Analysis and Statistics

1. **Collection, Organization and Presentation of Data** - Students collect and explore data, organize data into a useful form, and develop skill in representing and reading data displayed in different formats.
2. **Description and Interpretation** - Students examine data and describe characteristics of a distribution, relate data to the situation from which they arose, and use data to answer questions convincingly and persuasively.
3. **Inference and Prediction** - Students draw defensible inferences about unknown outcomes, make predictions, and identify the degree of confidence they have in their predictions.

IV. Number Sense and Numeration

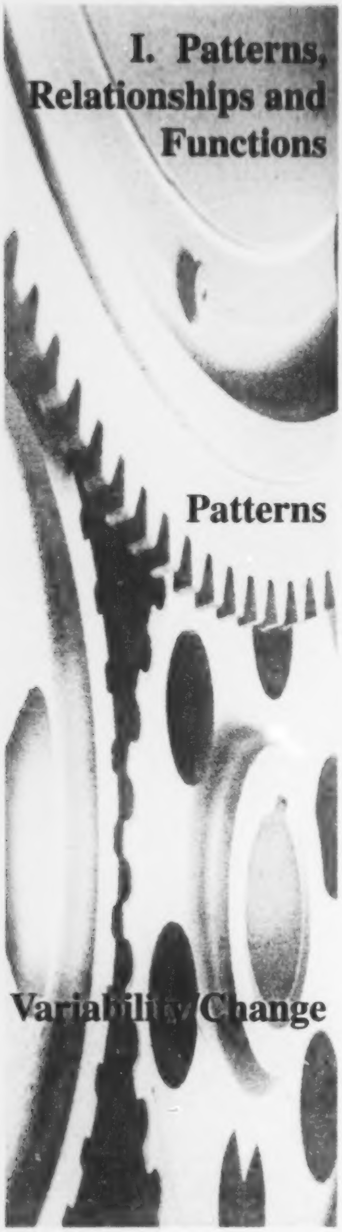
1. **Concepts and Properties of Numbers** - Students experience counting and measuring activities to develop intuitive sense about numbers, develop understanding about properties of numbers, understand the need for and existence of different sets of numbers, and investigate properties of special numbers.
2. **Representation and Uses of Numbers** - Students recognize that numbers are used in different ways such as counting, measuring, ordering and estimating, understand and produce multiple representations of a number, and translate among equivalent representations.
3. **Number Relationships** - Students investigate relationships such as equality, inequality, inverses, factors and multiples, and represent and compare very large and very small numbers.

V. Numerical and Algebraic Operations and Analytical Thinking

1. **Operations and their Properties** - Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems.
2. **Algebraic and Analytic Thinking** - Students analyze problems to determine an appropriate process for solution, and use algebraic notations to model or represent problems.

VI. Probability and Discrete Mathematics

1. **Probability** - Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available, and make critical judgments about claims that are made in probabilistic situations.
2. **Discrete Mathematics** - Students investigate practical situations such as scheduling, routing, sequencing, networking, organizing and classifying, and analyze ideas like recurrence relations, induction, iteration, and algorithm design.



I. Patterns, Relationships and Functions

Patterns, relationships and functions comprise one of the most important themes in the study of mathematics. Mathematical thinking begins with the recognition of similarities among objects or events, proceeds to generalization and abstraction, and culminates in the ability to understand, explain and make predictions. Contexts that exhibit structure and regularity provide rich opportunities for describing the physical world, studying mathematics and solving problems.

Patterns

Wherever there is mathematics there are patterns, and wherever there are patterns there is mathematics. Patterns are regularities or similarities that characterize sets of numbers, shapes, graphs, tables or other mathematical objects. Mathematicians look for patterns in everything they do; thus, mathematics is frequently defined as *the science of patterns.* In studying mathematics, students learn to recognize, describe, analyze and create patterns, to extend and generalize patterns, to create mathematical models based on observed patterns, and to predict the behavior of real-world phenomena based on such observed patterns. They learn to communicate the nature of mathematical patterns and relationships in various ways including words, physical models, diagrams, tables, charts, graphs, and equations. Since each representation highlights different aspects of the patterns and relationships, students must be able to construct multiple representations of mathematical relationships and to translate among them.

Variability/Change

Variability and change are as fundamental to mathematics as they are to the physical world, and an understanding of the concept of a *variable* is essential to mathematical thinking. Students must be able to describe the relationships among variables, to predict what will happen to one variable as another variable is changed, and to compare different patterns of change. The study of variability and change provides a basis for making sense of the world and of mathematical ideas.

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Strand I. Patterns, Relationships and Functions

Content Standard 1: Students recognize similarities and generalize patterns, use patterns to create models and make predictions, describe the nature of patterns and relationships, and construct representations of mathematical relationships. **(Patterns)**

	Elementary School	**Middle School**	**High School**
1. Recognizing, describing and generalizing patterns is the starting point of mathematics.	① Recognize, describe and extend numerical and geometric patterns.	① Describe, analyze and generalize patterns arising in a variety of contexts and express them in general terms.	① Analyze and generalize mathematical patterns including sequences, series, and recursive patterns.
2. Patterns and relationships are represented and communicated in diverse ways.	② Represent and record patterns and relationships in a variety of ways including tables, charts and pictures.	② Represent and record patterns in a variety of ways including tables, charts and graphs, and translate between various representations.	② Analyze, interpret and translate among representations of patterns including tables, charts, graphs, matrices and vectors.
3. Patterns enable students to describe and understand the physical world and to make informed predictions.	③ Use patterns to describe real-world phenomena.	③ Use patterns and their generalizations to make and justify inferences and predictions.	③ Study and employ mathematical models of patterns to make inferences, predictions and decisions.
4. Recognizing and classifying families of patterns enables students to understand and use their mathematical properties.	④ Explore various types of numeric and geometric patterns (repeating, growing, shrinking).	④ Explore and describe visual and numeric patterns, including linear expressions, near-linear patterns, and symmetric and spatial patterns	④ Explore patterns (graphic, numeric, etc.) characteristic of families of functions; explore structural patterns within systems of objects, operations or relations.
5. Pattern recognition and analysis provide an important key to solving problems and learning new mathematics.	⑤ Apply their experiences with patterns to help solve problems and explore new content.	⑤ Use patterns and generalizations to solve problems and explore new content.	⑤ Use patterns and reasoning to solve problems and explore new content.

Strand I. Patterns, Relationships and Functions

Content Standard 2: Students describe the relationships among variables, predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change. **(Variability and Change)**

	Elementary School	Middle School	High School
1. Studying change and variability in physical and abstract contexts is an important objective of mathematics.	① Recognize change and variability when it occurs in a variety of settings	① Identify and describe the nature of change; recognize change in more abstract and complex situations and explore different kinds of change, and patterns of variation.	① Identify and describe the nature of change and begin to use the more formal language such as rate of change, continuity, limit, distribution and deviation.
2. Variability becomes understandable when students recognize patterns of change and natural variation	② Recognize that change is often predictable, but variable, and that patterns emerge that help describe the change.	② Connect an initial state to a final state and generalize a rule that describes a pattern of change.	② Develop a mathematical concept of function and recognize that functions display characteristic patterns of change (e.g., linear, quadratic, exponential).
3. Changes are frequently interdependent; understanding patterns of change in one variable can help students predict changes in another.	③ Explore change, and realize that changes are frequently interdependent.	③ Begin to investigate applications in bivariate data and linear relationships, and explore questions of what will happen to one quantity if another variable is changed.	③ Expand their understanding of function to include non-linear functions, composition of functions, inverses of functions, and piece-wise and recursively-defined functions.
4. Variability is represented in a variety of symbolic forms.	④ Use tables, charts, open sentences and hands-on models to represent change and variability.	④ Represent variability or change by ordered pairs, tables, graphs and equations.	④ Represent functions using symbolism such as matrices, vectors, and functional representation $(f(x))$.
5. Functions and relationships are used to model patterns of variability arising from physical and mathematical contexts.	⑤ Begin to describe and differentiate between types of relationships, especially repeating, growing and shrinking patterns.	⑤ Differentiate between functions and relationships such as linear vs. not linear or continuous vs. non-continuous.	⑤ Differentiate and analyze classes of functions including linear, power, quadratic, exponential, circular and trigonometric functions and realize that many different situations can be modeled by a particular type of function.
6. Understanding variability and change is a basis for making sense of the world and of mathematical ideas.	⑥ Explore variability and change in a variety of contexts, investigations, and problems.	⑥ Continue to explore relationships arising from interesting contexts, and use variables and relationships to solve mathematical problems.	⑥ Increase their use of functions and mathematical models to solve problems in context.

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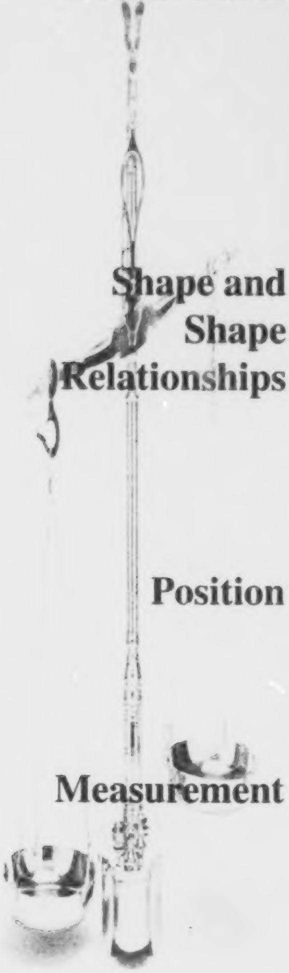
II. Geometry and Measurement



We live in a three-dimensional world. In order to interpret and make sense of that world, students need both analytical and spatial abilities. Geometry and measurement, which involve notions of shape, size, position, and dimension, are used extensively to describe and understand the world around us.

Shape and Shape Relationships

Spatial sense is developed when students recognize, draw, construct, visualize, compare, classify and transform geometric shapes in both two and three dimensions. They learn to identify those characteristics that are necessary to define a given shape, and they can differentiate one shape from another. Students also develop an awareness of the properties of a shape and of the relationships among shapes. This includes hierarchical classifications of shapes (e.g., all squares are rhombuses), relationships among components of a shape (e.g., opposite sides of a rectangle are parallel), symmetries of a shape, congruence and similarity.

Position

Position refers to the location of physical objects or points in space as well as to the relative locations and positions of objects, points, lines, planes and other geometric elements. It includes such notions as betweenness, collinearity and coordinates in two and three dimensions, as well as the locus of a point as it moves through space and the location of special points.

Measurement

Measurement reflects the usefulness and practicality of mathematics and puts students in touch with the world around them. Measurement requires the comparison of an attribute (distance, surface, capacity, mass, time, temperature) between two objects or to a known standard, the assignment of a number to represent the comparison, and the interpretation of the results. Measurement also introduces students to the important concepts of precision, approximation, tolerance, error and dimension.

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Strand II. Geometry and Measurement

Content Standard 1: Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes, identify properties and describe relationships among shapes. **(Shape and Shape Relationships)**

	Elementary School	Middle School	High School
1. Spatial sense relies on the ability to recognize and describe shape.	① Recognize and name familiar shapes in one, two and three dimensions, such as lines, rectangles and spheres, and informally discuss the shape of a graph.	① Distinguish among shapes and differentiate between examples and non-examples of shapes based on their properties; generalize about shapes of graphs and data distributions.	① Use shape to identify plane and solid figures, graphs, loci, functions, and data distributions.
2. Recognizing attributes and characteristics of shapes is a prerequisite for understanding.	② Describe the attributes of familiar shapes.	② Generalize the characteristics of shapes and apply their generalizations to classes of shapes.	② Determine necessary and sufficient conditions for the existence of a particular shape and apply those conditions to analyze shapes.
3. Comparing, sorting and classifying shapes leads to useful generalizations.	③ Compare, sort and classify familiar shapes.	③ Derive generalizations about shapes and apply those generalizations to develop classifications of familiar shapes.	③ Use transformational, coordinate or synthetic methods to verify (prove) the generalizations they have made about properties of classes of shapes.
4. Drawing and constructing shapes in two and three dimensions are important ways to represent the world.	④ Draw and build familiar shapes.	④ Construct familiar shapes using coordinates, appropriate tools (including technology), sketching and drawing two- and three-dimensional shapes.	④ Draw and construct shapes in two- and three-dimensions and analyze and justify the steps of their constructions.
5. Understanding shapes requires recognition of what happens when shapes are combined, dissected or transformed.	⑤ Explore ways to combine, dissect and transform shapes.	⑤ Combine, dissect and transform shapes.	⑤ Study transformations of shapes using isometries, size transformations and coordinate mappings.
6. Figures that are alike in size and/or shape and figures that have special relationships to each other lead to important generalizations.	⑥ Recognize parallel and perpendicular line segments and figures that have similarity and/or congruence.	⑥ Generalize about the common properties of similar, congruent, parallel and perpendicular shapes and verify their generalizations informally.	⑥ Compare and analyze shapes and formally establish the relationships among them, including congruence, similarity, parallelism, perpendicularity and incidence.
7. Shape, shape properties, and shape relationships help students to describe and make sense of the physical world and to solve problems.	⑦ Use shape, shape properties and shape relationships to describe the physical world and to solve problems.	⑦ Use shape, shape properties and shape relationships to describe the physical world and to solve problems.	⑦ Use shape, shape properties and shape relationships to describe the physical world and to solve problems.

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Strand II. Geometry and Measurement

Content Standard 2: Students identify locations of objects, identify location relative to other objects, and describe the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object. **(Position)**

	Elementary School	**Middle School**	**High School**
1. Locating physical objects or points in space requires understanding of position.	① Locate and describe objects in terms of their position, including front, back, inside, outside, right, left, over, under, next to, between, and locations on the number line, on a coordinate graph and on a map.	① Locate and describe objects in terms of their position, including compass directions, Cartesian coordinates, latitude and longitude, and midpoints.	① Locate and describe objects in terms of their position, including polar coordinates, three-dimensional Cartesian coordinates, vectors, and limits.
2. Concepts of direction, orientation , relative position and symmetry enable students to describe objects relative to their surroundings.	② Locate and describe objects in terms of their orientation, direction and relative position, including up, down, front, back, N-S-E-W, flipped, turned, translated; recognize symmetrical objects and identify their lines of symmetry.	② Locate and describe objects in terms of their orientation and relative position, including coincident, collinear, parallel, perpendicular; differentiate between fixed (e.g., N-S-E-W) and relative (e.g., right-left) orientations; recognize and describe examples of bilateral and rotational symmetry.	② Locate and describe objects in terms of their orientation and relative position, including displacement (vectors), phase shift, maxima, minima and inflection points; give precise mathematical descriptions of symmetries.
3. Certain actions can change the size, shape, position or orientation of an object.	③ Explore what happens to the size, shape and position of an object after sliding, flipping, turning, enlarging or reducing it.	③ Describe translations, reflections, rotations and dilations using the language of transformations, and employ transformations to verify congruence of figures.	③ Give precise mathematical descriptions of transformations and describe the effects of transformations on size, shape, position and orientation.
4. Locating all the points that satisfy a condition or the special points that satisfy two or more conditions is an important spatial ability.		④ Locate the position of points or objects described by two or more conditions; locate all the points (locus) that satisfy a given condition.	④ Describe the locus of a point by a rule or mathematical expression; trace the locus of a moving point.
5. Concepts of position, direction and orientation enable students to describe the physical world and to solve problems.	⑤ Use concepts of position, direction and orientation to describe the physical world and to solve problems.	⑤ Use concepts of position, direction and orientation to describe the physical world and to solve problems.	⑤ Use concepts of position, direction and orientation to describe the physical world and to solve problems.

Strand II. Geometry and Measurement

Content Standard 3: Students compare attributes of two objects, or of one object with a standard (unit), and analyze situations to determine what measurement(s) should be made and to what level of precision. **(Measurement)**

	Elementary School	**Middle School**	**High School**
1. A fundamental component of measurement and learning to measure is the comparison of an object or property to a unit of comparison	① Compare attributes of objects; develop standard units of measurement; and select and use standard tools for measurement.	① Select and use appropriate tools; measure objects using standard units in both the metric and common systems, and measure angles in degrees.	① Select and use appropriate tools; make accurate measurements using both metric and common units, and measure angles in degrees and radians.
2. Measurement requires that students identify the attribute to be measured and an appropriate unit.	② Identify the attribute to be measured and select the appropriate unit of measurement for length, mass (weight), area, perimeter, capacity, time, temperature and money.	② Identify the attribute to be measured and select the appropriate unit of measurement for length, mass (weight), time, temperature, perimeter, area, volume, angle.	② Continue to make and apply measurements of length, mass (weight), time, temperature, area, volume, angle; classify objects according to their dimensions.
3. Students develop a better understanding of the physical world if they regularly estimate before they measure and evaluate their estimates after they measure.	③ Develop strategies for estimating measures and compare the estimates to the results of the measurement; decide if an estimate is "a good estimate."	③ Estimate measures with a specified degree of accuracy and decide if an estimate or a measurement is "close enough."	③ Estimate measures with a specified degree of accuracy and evaluate measurements for accuracy, precision and tolerance.
4. Measurement is incomplete unless students interpret the meaning and significance of their results.	④ Explain the meaning of measurements and recognize that the number of units it takes to measure an object is related to the size of the unit.	④ Interpret measurements and recognize that two objects may have the same measurement on one attribute (e.g., area), but not necessarily on another (e.g., perimeter).	④ Interpret measurements and explain how changes in one measure may affect other measures.
5. It is not always possible to measure a quantity directly; in such cases students must use other indirect means.	⑤ Explore scale drawings, models and maps and relate them to measurements of real objects.	⑤ Use proportional reasoning and indirect measurements to draw inferences.	⑤ Use proportional reasoning and indirect measurements, including applications of trigonometric ratios, to measure inaccessible distances and to determine derived measures such as density.
6. Measurement reflects the usefulness and practicality of mathematics and puts students in touch with the physical world.	⑥ Apply measurement to describe the real world and to solve problems.	⑥ Apply measurement to describe the real world and to solve problems.	⑥ Apply measurement to describe the real world and to solve problems.

III. Data Analysis and Statistics

We live in a sea of information. In order not to drown in the data that inundate our lives every day, we must be able to process and transform data into useful knowledge. The ability to interpret data and to make predictions and decisions based on data is an essential basic skill for every individual.

Collection, Organization and Presentation of Data

Knowing what data to collect and where and how to collect them is the starting point of quantitative literacy. The mathematics curriculum should capitalize on students' natural curiosity about themselves and their surroundings to motivate them to collect and explore interesting statistics and measurements derived from both real and simulated situations. Once the data are gathered, they must be organized into a useful form, including tables, graphs, charts and pictorial representations. Since different representations highlight different patterns within the data, students should develop skill in representing and reading data displayed in different formats, and they should discern when one particular representation is more desirable than another.

Description and Interpretation

Students must be able to examine data and describe salient characteristics of the distribution. They also must be able to relate the data to the physical situation from which they arose. Students should use the data to answer key questions and to convince and persuade.

Inference and Prediction

Based on known data, students should be able to draw defensible inferences about unknown outcomes. They should be able to make predictions and to identify the degree of confidence that they place in their predictions.

Strand III. Data Analysis and Statistics

Content Standard 1: Students collect and explore data, organize data into a useful form, and develop skill in representing and reading data displayed in different formats. **(Collection, Organization and Presentation of Data)**

	Elementary School	**Middle School**	**High School**
1. Data drive many facets of modern society; knowing what data to collect and when and how to obtain them is the starting point of quantitative literacy.	① Collect and explore data through counting, measuring and conducting surveys and experiments.	① Collect and explore data through observation, measurement, surveys, sampling techniques and simulations.	① Collect and explore data through observation, measurement, surveys, sampling techniques and simulations.
2. Data are of little use until they are organized and presented in a meaningful format.	② Organize data using concrete objects, pictures, tallies, tables, charts, diagrams and graphs.	② Organize data using tables, charts, graphs, spreadsheets and data bases.	② Organize data using tables, charts, graphs, spreadsheets and data bases.
3. Since different representations highlight different patterns in the data, students must make critical judgments.	③ Present data using a variety of appropriate representations, and explain the meaning of the data	③ Present data using a variety of appropriate representations, and explain why one representation is preferred over another or how a particular representation may bias the presentation.	③ Present data using the most appropriate representation and give a rationale for their choice; show how certain representations may skew the data or bias the presentation.
4. To solve problems, students frequently must decide what data are needed and plan how to obtain, organize and present them.	④ Identify what data are needed to answer a particular question or solve a given problem, and design and implement strategies to obtain, organize and present those data.	④ Identify what data are needed to answer a particular question or solve a given problem, and design and implement strategies to obtain, organize and present those data.	④ Identify what data are needed to answer a particular question or solve a given problem, and design and implement strategies to obtain, organize and present those data.

Strand III. Data Analysis and Statistics

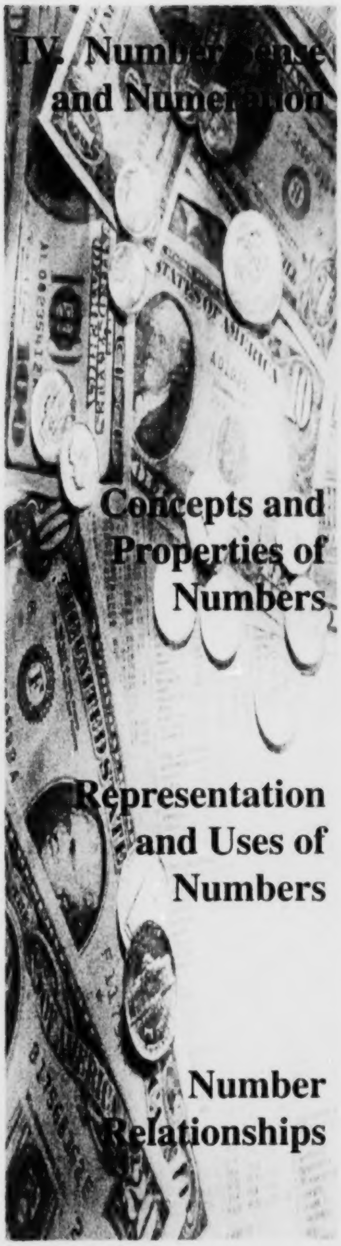
Content Standard 2: Students examine data and describe characteristics of the distribution, relate data to the situation from which they arose, and use data to answer questions convincingly and persuasively.
(Description and Interpretation)

	Elementary School	**Middle School**	**High School**
1. The ability to read and interpret data has become a basic-literacy skill in today's world	① Read and explain data they have collected and organized themselves and progress to reading data from other sources.	① Critically read data from tables, charts or graphs and explain the source of the data and what the data represent.	① Critically read data from tables, charts or graphs and explain the source of the data and what the data represent.
2. Patterns in data distributions help students to interpret the findings.	② Describe the shape of the data using informal language	② Describe the shape of a data distribution and identify the center, the spread, correlations, and any outliers.	② Describe the shape of a data distribution and determine measures of central tendency, variability and correlation.
3. Students learn to draw conclusions and to convince and persuade using data to justify their positions.	③ Draw, explain and justify conclusions such as trends based on data	③ Draw, explain and justify conclusions based on data	③ Use the data and their characteristics to draw and support conclusions.
4. Students should think critically about the data they encounter and exercise judgment in describing and interpreting data.	④ Raise and answer questions about the source, collection, organization and presentation of data as well as the conclusions drawn from the data; explore biases in data.	④ Critically question the sources of data; the techniques used to collect, organize and present data; the inferences drawn from the data; and the possible sources of bias in the data or their presentation.	④ Critically question the sources of data; the techniques used to collect, organize and present data; the inferences drawn from the data; and the sources of bias and measures taken to eliminate such bias.
5. Gathering and interpreting data are important strategies for analyzing and solving problems.	⑤ Formulate questions and problems, and gather and interpret data to answer those questions.	⑤ Formulate questions and problems, and gather and interpret data to answer those questions.	⑤ Formulate questions and problems, and interpret data to answer those questions.

Strand III. Data Analysis and Statistics

Content Standard 3: Students draw defensible inferences about unknown outcomes, make predictions, and identify the degree of confidence they have in their predictions. **(Inference and Prediction)**

	Elementary School	**Middle School**	**High School**
1. Making and testing hypotheses is an essential technique for gaining new knowledge.	① Make and test hypotheses.	① Make and test hypotheses.	① Make and test hypotheses.
2. In order to test hypotheses, students must carefully design their experimental techniques.	② Conduct surveys, samplings and experiments to solve problems and answer questions of interest to them.	② Design experiments to model and solve problems using sampling, simulations, and controlled investigations.	② Design investigations to model and solve problems; also employ confidence intervals and curve fitting in analyzing the data.
3. Critical judgment develops as students learn to formulate, communicate and evaluate arguments and conclusions based on data.	③ Formulate and communicate arguments and conclusions based on data and evaluate their arguments and those of others.	③ Formulate and communicate arguments and conclusions based on data and evaluate their arguments and those of others.	③ Formulate and communicate arguments and conclusions based on data and evaluate their arguments and those of others.
4. Patterns in known data give students confidence in making inferences about unknown situations.	④ Make and explain predictions based on data.	④ Make predictions and decisions based on data, including interpolations and extrapolations.	④ Make predictions and decisions based on data, including interpolations and extrapolations.
5. Students learn that inferences and predictions are powerful tools for answering questions and solving problems.	⑤ Make predictions to answer questions and solve problems.	⑤ Employ investigations, mathematical models and simulations to make inferences and predictions to answer questions and solve problems.	⑤ Employ investigations, mathematical models and simulations to make inferences and predictions to answer questions and solve problems.



IV. Number Sense and Numeration

Number sense is to mathematics what vocabulary is to language. Students must learn to quantify and measure, concretely at first and increasingly more abstractly as they mature. They also must develop an understanding of numeration systems and of the structure of such systems. They must learn to estimate mathematical quantities and to represent and communicate mathematical ideas in the language of mathematics.

Concepts and Properties of Numbers

Fundamental questions like *What is a number?* or *What is three?* can be deceptively difficult to answer. Students require extensive involvement with concrete experiences of counting and measuring in order to develop an intuitive sense about number. Through both informal and formal means, students develop understanding about important properties of numbers such as even vs. odd, whole number vs. fraction, positive vs. negative. They understand the existence of different sets of numbers (whole numbers, integers, rationals, reals, ...) and the properties of special numbers such as 0, 1, π, or the inverse of a number.

Representation and Uses of Numbers

Students recognize that numbers are used in different ways such as to answer the questions "How many?" (counting), "How much?" (measuring), and "Which one?" (ordering). They understand that a numerical quantity can be represented in many different ways, and they can produce multiple representations of numbers (e.g., fractions, decimals, and per cents in the middle grades; vectors and coordinate representations in later years) and they can translate easily among equivalent representations. As students mature from the middle school on, they develop a solid understanding of both linearity and proportionality.

Number Relationships

Students develop understanding of important relationships among numbers including the relationships of = and ≠ (<, >); of opposites (additive inverses) and reciprocals (multiplicative inverses); of factors and multiples; of primes, composites, and relatively prime numbers; of powers and roots. They understand and can represent very large and very small numbers and can compare the orders of magnitude of numbers.

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Strand IV. Number Sense and Numeration

Content Standard 1: Students experience counting and measuring activities to develop intuitive sense about numbers, develop understanding about properties of numbers, understand the need for and existence of different sets of numbers, and investigate properties of special numbers. **(Concepts and Properties of Numbers)**

	Elementary School	**Middle School**	**High School**
1. An intuitive quantitative sense develops from students' investigations of numbers and their properties.	① Develop an understanding of whole numbers, and read, write and count using whole numbers; investigate basic concepts of fractions and decimals.	① Develop an understanding of integers and rational numbers, and represent rational numbers in both fraction and decimal form.	① Develop an understanding of irrational, real and complex numbers.
2. A solid understanding of the numeration system is essential for later success with calculations.	② Investigate and develop an understanding of the base-10 place-value system.	② Extend their understanding of numeration systems to include decimal numeration, scientific numeration and non-decimal numeration systems.	② Use the (a+bi) and polar forms of complex numbers.
3. Important properties provide students with deeper insight into numbers and their uses.	③ Develop an understanding of the properties of numbers (e.g., order) and of the properties of the special numbers 0 and 1.	③ Develop an understanding of the properties of the integer and rational number systems (e.g., order, density) and of the properties of special numbers including 0, 1, π, and the additive and multiplicative inverses.	③ Develop an understanding of the properties of the real and complex number systems and of the properties of special numbers including π, i, e, and conjugates.
4. Numeration systems become most useful as students use them to model and describe problems.	④ Apply their understanding of number systems to model and solve problems.	④ Apply their understanding of number systems to model and solve mathematical and applied problems.	④ Apply their understanding of number systems to model and solve mathematical and applied problems.

Strand IV. Number Sense and Numeration

Content Standard 2: Students recognize that numbers are used in different ways such as counting, measuring, ordering, and estimating, understand and produce multiple representations of a number, and translate among equivalent representations. **(Representation and Uses of Numbers)**

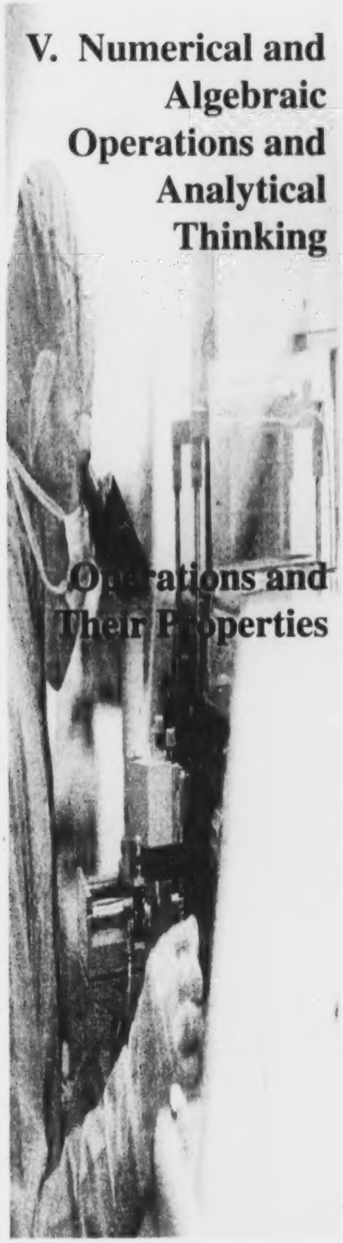
	Elementary School	Middle School	High School
1. Students recognize and understand numbers that they encounter in varied formats.	① Represent whole numbers, fractions and decimals using concrete, pictorial and symbolic representations.	① Give geometric representations of fractions, prime and composite numbers, triangular and square numbers, and other number concepts; represent rational numbers and integers on the number line.	① Give decimal representations of rational and irrational numbers and coordinate and vector representations of complex numbers.
2. Numeracy requires that students recognize when numbers are equivalent even though they may be represented in different formats.	② Explore and recognize different representations for the same number and explain why they are the same.	② Recognize equivalent representations of a number, especially fractions, decimals and percents, and translate freely among representations.	② Develop an understanding of more complex representations of numbers, including exponential and logarithmic expressions, and select an appropriate representation to facilitate problem solving.
3. Numbers are used for varied purposes, and it is important to differentiate among their uses.	③ Investigate ways numbers are used (e.g., counting, ordering, naming, locating, measuring).	③ Distinguish between numbers that are used for counting, numbers that are used for ordering, numbers that are used for measuring, and numbers that are used for naming.	③ Determine when to use rational approximations and the exact values of numbers such as e, π and the irrationals.
4. Estimation is one of the most important skills for students to develop and use on a regular basis.	④ Develop strategies for estimating quantity and evaluate the reasonableness of their estimates.	④ Develop and refine strategies for estimating quantities, including fractional quantities, and evaluate the reasonableness and appropriateness of their estimates.	④ Apply estimation in increasingly complex situations.
5. Knowing what numbers to use and how to represent them is key to students' abilities to solve problems.	⑤ Select appropriate numbers and representations in order to solve problems.	⑤ Select appropriate representations for numbers, including integers and rational numbers, in order to simplify and solve problems.	⑤ Select appropriate representations for numbers, including representation of rational and irrational numbers and coordinate or vector representation of complex numbers, in order to simplify and solve problems.

Strand IV. Number Sense and Numeration

Content Standard 3: Students investigate relationships such as equality, inequality, inverses, factors and multiples, and represent and compare very large and very small numbers. **(Number Relationships)**

	Elementary School	Middle School	High School
1. Relationships of equality and inequality are among the most fundamental in mathematics.	① Compare and order numbers using "equal," "less than" or "greater than."	① Compare and order integers and rational numbers using relations of equality and inequality.	① Compare and order real numbers and compare rational approximations to exact values.
2. Students learn the importance of making comparisons between numbers, especially as ratios and rates.	② Use part-whole relationships to explore numbers, develop number concepts, and understand computation.	② Express numerical comparisons as ratios and rates.	② Express numerical comparisons as ratios and rates.
3. By classifying numbers according to their properties and identifying important numerical relationships, students develop a deeper understanding of numbers.	③ Classify numbers as even or odd and explore concepts of factors and multiples.	③ Distinguish between prime and composite numbers; identify factors, multiples, common factors and multiples, and relatively prime numbers; and apply divisibility tests to numbers.	③ Extend the relationships of primes, factors, multiples and divisibility in an algebraic setting.
4. Numbers that are related exponentially exhibit important relationships that students will encounter in a variety of applications.		④ Explain the meaning of powers and roots of numbers and use calculators to compute powers and square roots.	④ Express number relationships using positive and negative rational exponents, logarithms, and radicals.
5. Students can invoke important number relationships to help them understand and solve problems.	⑤ Apply their understanding of number relationships in solving problems.	⑤ Apply their understanding of number relationships in solving problems.	⑤ Apply their understanding of number relationships in solving problems

V. Numerical and Algebraic Operations and Analytical Thinking



The ability to represent quantitative situations with algebraic symbolism, numerical operations and algebraic thinking is essential to solving problems in significant contexts and applications. The concepts of *number* and *variable* and their symbolic representation and manipulation are central to the understanding of arithmetic and its generalization in algebra. The contemporary applications of mathematics in virtually every field of work and study rely on algebraic and analytic thinking and communication as fundamental tools.

Operations and Their Properties

The ultimate reason for mastering the operations of arithmetic and algebra is to solve problems. To that end, understanding the basic computational operations and their algorithms is essential for competence in mathematics, but the emphasis must be on understanding and using the operations, not on memorizing algorithms. In computation, understanding and accuracy are always more important than speed. Understanding the operations requires the concomitant understanding and application of the properties of those operations, and it involves knowing what operations to use in a particular situation.

There is no one way to perform a calculation. Students must be competent in performing calculations, but they need not have a rigid adherence to one algorithm. Methods of computation include proficiency with mental calculation, paper and pencil, and calculators; the ability to represent computations with manipulatives and geometric models; and the discernment of which computational method to use in a given situation. Computational methods also involve estimating and assessing the reasonableness of the results of a computation.

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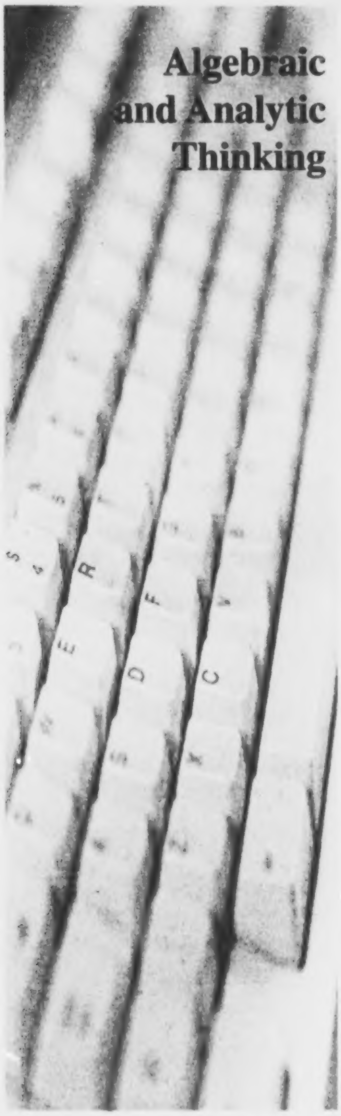

Algebraic and Analytic Thinking

Mathematical representations allow us to visualize and understand problems. These representations may be numerical, literal, symbolic, graphical, pictorial or physical. Facility with multiple representations of numerical and algebraic concepts and relationships is essential to mathematical competence. This includes the development of "symbol sense" as well as "number sense" and the understanding that the notion of *solution* involves a process as well as a product. Thus, the solution of a mathematical problem requires both an understanding of the question for which an answer is sought and the development of a strategy to obtain that answer. The context of the problem determines the nature and the degree of precision of the required solution.

The increasing use of quantitative methods in all disciplines has made algebra the fundamental tool for mathematical applications. Algebraic thinking is learned most effectively when it is studied in the context of applications, both mathematical and real-world, that reveal the power of algebra to model real problems and to generalize to new situations. Students should use algebraic techniques to analyze and describe relationships, to model problem situations, and to examine the structure of mathematical relationships. The algebra curriculum should employ contemporary technology, including spreadsheets and graphical analysis, to emphasize conceptual understanding of algebra and analytic thinking as sophisticated means of representation and as powerful problem-solving tools.

Strand V. Numerical and Algebraic Operations and Analytical Thinking

Content Standard 1: Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems. **(Operations and their Properties)**

	Elementary School	**Middle School**	**High School**
1. Understanding the basic computational operations is essential for competence in mathematics, but there is no one way to perform a calculation.	① Use manipulatives to model operations with numbers; develop their own methods of recording operations; and relate their models and recordings to standard symbolic expressions and algorithms.	① Use manipulatives and diagrams to model operations and their inverses with integers and rational numbers and relate the models to their symbolic expressions.	① Present and explain geometric and symbolic models for operations with real and complex numbers, and algebraic expressions.
2. Methods of computation include proficiency with mental calculation, paper and pencil, and calculators; students must know which method is most appropriate for a given task.	② Develop and apply the appropriate method of computation from among mental computation, estimation, paper-and-pencil or calculators; explain why they are choosing a method and how they know which operations to perform in a given situation.	② Compute with integers, rational numbers and simple algebraic expressions using mental computation, estimation, calculators, and paper-and-pencil; explain what they are doing and know which operations to perform in a given situation.	② Compute with real numbers, complex numbers, algebraic expressions, matrices and vectors using technology and, for simple instances, with paper-and-pencil algorithms.
3. Understanding the operations requires that students also understand the properties of those operations and how to apply them.	③ Explore properties of operations (e.g., commutative and distributive properties) and give examples of how they use those properties.	③ Describe the properties of operations with rationals and integers (e.g., closure; associative, commutative and distributive properties) and give examples of how they use those properties.	③ Describe the properties of operations with numbers, algebraic expressions, vectors and matrices and make generalizations about the properties of given mathematical systems.
4. The ultimate reason for mastering the computational operations and their algorithms is to solve problems.	④ Apply operations efficiently and accurately in solving problems.	④ Efficiently and accurately apply operations with integers, rational numbers, and simple algebraic expressions in solving problems.	④ Efficiently and accurately apply operations with real numbers, complex numbers, algebraic expressions, matrices, and vectors in solving problems.

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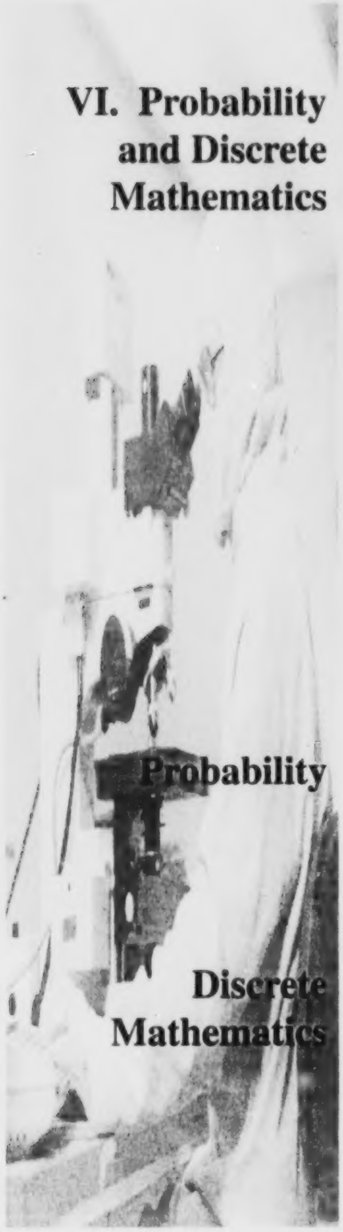
Strand V. Numerical and Algebraic Operations and Analytical Thinking

Content Standard 2: Students analyze problems to determine an appropriate process for solution, and use algebraic notations to model or represent problems. **(Algebraic and Analytic Thinking)**

	Elementary School	Middle School	High School
1. Students develop both symbol sense and number sense as they learn to read, write and speak the language of mathematics.	① Write and solve open sentences (e.g. $\Diamond + \Delta = 5$) and write stories to fit the open sentence.	① Read and write algebraic expressions; develop original examples expressed verbally and algebraically; simplify expressions and translate between verbal and algebraic expressions; and solve linear equations and inequalities.	① Identify important variables in a context, symbolize them, and express their relationships algebraically.
2. Mathematical representations, which may be numerical, literal, symbolic, graphical, pictorial or physical, enable students to visualize and understand problems.	② Explore algebraic concepts with manipulatives such as balance scales, tables of input and output, and pictorial representations of problems.	② Represent algebraic concepts with geometric models (e.g., algebra tiles), physical models (e.g., balance beam), tables and graphs; and write algebraic expressions to correspond to the multiple representations.	② Represent algebraic concepts and relationships with matrices, spreadsheets, diagrams, graphs, tables, physical models, vectors, equations and inequalities; and translate among the various representations.
3. Solving mathematical problems involves a process as well as a product; the context of the problem determines the nature of the solution.	③ Find replacements for the variable(s) in open sentences.	③ Solve linear equalities and inequalities using algebraic and geometric methods, and use the context of the problem to interpret and explain their solutions.	③ Solve linear equations and inequalities algebraically and non-linear equations using graphing, symbol-manipulating or spreadsheet technology; and solve linear and non-linear systems using appropriate methods.
4. Students learn analytic thinking most effectively when it is studied in the context of problems and applications.	④ Use analytic thinking to describe situations and solve problems.	④ Analyze problems modeled by linear functions, determine strategies for solving the problems, and evaluate the adequacy of the solutions in the context of the problems.	④ Analyze problems that can be modeled by functions, determine strategies for solving the problems, and evaluate the adequacy of the solutions in the context of the problems.
5. Students employ algebraic and analytic thinking and the power of technology to explore problems that reveal the many ways that mathematics is used in a wide variety of contemporary applications.		⑤ Explore problems that reflect the contemporary uses of mathematics in significant contexts and use the power of technology and algebraic and analytic reasoning to experience the ways mathematics is used in society.	⑤ Explore problems that reflect the contemporary uses of mathematics in significant contexts and use the power of technology and algebraic and analytic reasoning to experience the ways mathematics is used in society.

VI. Probability and Discrete Mathematics



Contemporary uses of mathematics demand that students learn to deal with uncertainty, to make informed decisions based on evidence and expectations, to exercise critical judgment about conclusions drawn from data, and to apply mathematical models to real-world phenomena. The technological world in which we live also depends upon information and the communication of information and upon applications of systems with separate (discrete) entities. Topics of discrete mathematics such as counting and permutation problems, matrix operations, vertex-edge networks, and relationships among finite sets have significant real-world applications that students will encounter in diverse fields of work and study.

Probability

Dealing with uncertainty and making predictions and decisions in the face of uncertainty are essential skills for coping with the modern world. Students must develop an understanding of the notion of uncertainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available. They also must be able to make critical judgments about claims that are made in probabilistic situations.

Discrete Mathematics

Discrete (discontinuous) mathematics has grown in significance in recent years and today has applications in many important practical situations such as scheduling, routing, sequencing, networking, organizing and classifying. Important ideas like recurrence relations, induction and algorithm design also have practical applications in a variety of fields. Computers, which are finite, discrete machines, require an understanding of discrete mathematics for the solution of problems using computer methods.

Strand VI. Probability and Discrete Mathematics

Content Standard 1: Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available, and make critical judgments about claims that are made in probabilistic situations. **(Probability)**

	Elementary School	Middle School	High School
1. Students develop an understanding of the concepts of chance and uncertainty.	① Explain the difference between chance and certainty and give examples to illustrate their understanding.	① Describe events as likely or unlikely and give qualitative and quantitative descriptions of the degree of likelihood.	① Develop an understanding of randomness and chance variation and describe chance and certainty in the language of probability.
2. Students express the likelihood of chance events in terms of probabilities.	② Compare events and describe them as "more likely" or "less likely" and use the language of fractions to describe simple probabilities.	② Describe probability as a measure of certainty ranging from 0 to 1, and conduct activities that allow them to express probabilities of simple events in mathematical terms.	② Give a mathematical definition of probability and determine the probabilities of more complex events, and generate and interpret probability distributions.
3. Through experiments students learn that some outcomes are affected by prior events, while others are independent.	③ Conduct experiments with concrete objects to explore concepts and develop an intuitive understanding of how the conditions of the experiment can affect the outcome.	③ Conduct experiments and give examples to illustrate the difference between dependent and independent events.	③ Analyze events to determine their dependence or independence and calculate probabilities of compound events.
4. Students also learn to examine outcomes and search for explanations, and they realize the difference between probabilities determined from observations and probabilities derived mathematically.	④ Conduct experiments, record the outcomes, examine those outcomes to determine if they make sense, and search for explanations of the outcomes.	④ Explain the difference between probabilities determined from experiments or chance events (empirical) and probabilities derived mathematically (theoretical), and explain how the empirical probability changes for a large number of trials.	④ Use sampling and simulations to determine empirical probabilities and, when appropriate, compare them to the corresponding theoretical probabilities; understand and apply the law of large numbers.
5. Making predictions and decisions in the face of uncertainty are essential skills for coping with the modern world.	⑤ Conduct probability experiments and simulations to model and solve problems.	⑤ Conduct probability experiments and simulations to model and solve problems.	⑤ Conduct probability experiments and simulations to model and solve problems, including compound events.

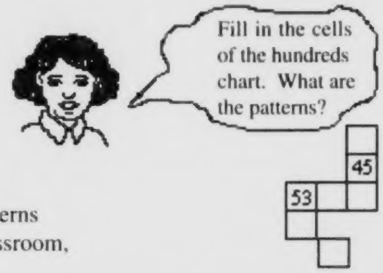
Strand VI. Probability and Discrete Mathematics

Content Standard 2: Students investigate practical situations such as scheduling, routing, sequencing, networking, organizing and classifying, and analyze ideas like recurrence relations, induction, iteration, and algorithm design. **(Discrete Mathematics)**

	Elementary School	**Middle School**	**High School**
1. Problems involving counting and arranging finite collections of objects occur in many applications.	① Use manipulatives and diagrams to explore problems involving counting and arranging objects.	① Use manipulatives, diagrams, and the fundamental theorem of counting to count permutations and combinations.	① Derive and use formulas for calculating permutations and combinations.
2. Concepts of sets and set relationships give students useful tools for representing problems.	② Explore sets and set relationships by sorting and classifying objects.	② Use sets and set relationships to explore and solve simple algebraic and geometric problems.	② Use sets and set relationships to represent algebraic and geometric concepts.
3. Many important practical applications involve networks.	③ Explore situations in which they model and trace paths using figures consisting of vertices connected by edges.	③ Solve problems involving networks, for example planning delivery routes or counting paths between points.	③ Use vertex-edge graphs to solve network problems such as finding circuits, critical paths, minimum spanning trees, and adjacency matrices.
4. Many important practical applications are modeled by recurrence relations.	④ Explore now-next patterns.	④ Explore recurrence relations and iterations.	④ Analyze and use discrete ideas such as induction, iteration and recurrence relations.
5. Mathematical applications frequently require students to develop their own procedures for solving problems	⑤ Explore, develop and invent their own algorithms to accomplish a task or to solve numerical problems.	⑤ Continue to use manipulatives and drawings to model the concepts and procedures for the standard arithmetic algorithms, and develop and analyze their own and other students' algorithms to accomplish a task or solve a mathematical problem.	⑤ Describe and analyze efficient algorithms to accomplish a task or solve a problem in a variety of contexts including practical, mathematical and computer-related situations.
6. Applications of discrete mathematics drawn from many important practical situations introduce students to contemporary uses of mathematics.	⑥ Use discrete mathematics concepts as described above to model situations and solve problems; and look for whether or not there is a solution (existence problems), determine how many solutions there are (counting problems), and decide upon a best solution (optimization problems).	⑥ Use discrete mathematics concepts as described above to model situations and solve problems; and look for whether or not there is a solution (existence problems), determine how many solutions there are (counting problems), and decide upon a best solution (optimization problems).	⑥ Use discrete mathematics concepts as described above to model situations and solve problems; and look for whether or not there is a solution (existence problems), determine how many solutions there are (counting problems), and decide upon a best solution (optimization problems).

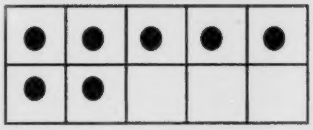
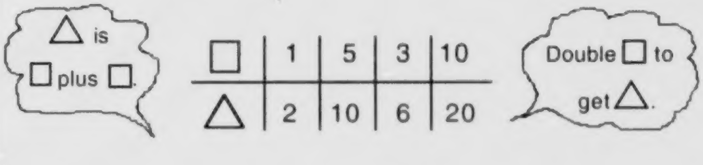
I. Patterns, Relationships and Functions

1. Students recognize similarities and generalize patterns; use patterns to create models and make predictions; describe the nature of patterns and relationships; and construct representations of mathematical relationships. (Patterns)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
1. Recognize, describe and extend numerical and geometric patterns.	• modeling patterns using objects such as counters, buttons, colored cubes.  Fill in the cells of the hundreds chart. What are the patterns? 45 53 • recording in pictures and describing in words patterns in various settings: - creating a journal of patterns seen at home, in the classroom, in a magazine, in data. - making and then describing a border design, a quilt pattern, etc. - explaining their tessellation to someone. • describing and analyzing patterns and relationships: - what is the pattern? - what are the relationships? • extending patterns, supplying missing elements in patterns and generalizing a rule to describe a pattern. • creating and explaining their own patterns.

I. Patterns, Relationships and Functions

1. Students recognize similarities and generalize patterns; use patterns to create models and make predictions; describe the nature of patterns and relationships; and construct representations of mathematical relationships. (Patterns)

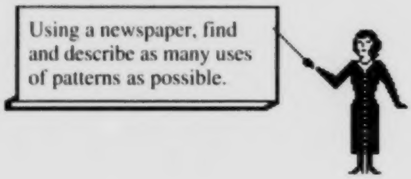
In the early years, children have the opportunity to...	***By engaging in activities such as...***
2. Represent and record patterns and relationships in a variety of ways including tables, charts and pictures.	• using sounds, motions, shapes, objects, pictures, and symbols to represent patterns and repetitions of events and designs in different ways. • exploring and comparing multiple ways of representing patterns and relationships such as using tables versus graphs. • using a ten frame as a visual pattern for developing basic facts and part-part-whole relationships and to explore numerical patterns and pattern relationships.  "I see 7 as 5 plus 2." "I see 7 plus 3 more is 10." "It takes 3 more to get to 10." • graphing information of importance to the children (e.g., spelling scores over time, growth of a plant, weather graph over time) as a means of looking for patterns, trends and relationships. • using tables, student-developed formulas, and graphs to show relationships which provide the foundation for describing and representing functions. - write a rule for a pattern.  △ is □ plus □. □ 1 5 3 10 △ 2 10 6 20 Double □ to get △.

I. Patterns, Relationships and Functions

1. Students recognize similarities and generalize patterns; use patterns to create models and make predictions; describe the nature of patterns and relationships; and construct representations of mathematical relationships. (Patterns)

<table>
<tr><th>In the early years, children have the opportunity to...</th><th>By engaging in activities such as...</th></tr>
<tr><td>3. Use patterns to describe real-world phenomena.</td><td>

- identifying patterns and regularity in the environment and in actions and events (e.g., quilt patterns, symmetry in leaves, border designs, fabric patterns, sequence of events, days of the week).



Using a newspaper, find and describe as many uses of patterns as possible.

- circling or coloring patterns in a hundreds chart to describe a numerical relationship:
  - diagonal is add eleven.
  - across you add one.
  - down you add ten.

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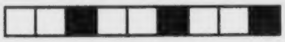
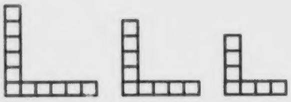
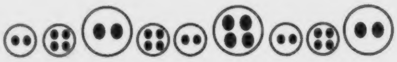
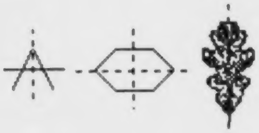
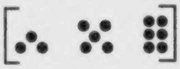
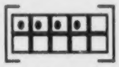
I. Patterns, Relationships and Functions

1. Students recognize similarities and generalize patterns; use patterns to create models and make predictions; describe the nature of patterns and relationships; and construct representations of mathematical relationships. (Patterns)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
4. Explore various types of numeric and geometric patterns (repeating, growing, shrinking).	• exploring different types and uses of patterns, progressing from relatively simple patterns to more complex patterns. For instance: - repeating patterns.  - growing patterns. 2, 4, 6, 8, 10, . . . - shrinking patterns.  - combination patterns.  - symmetry. 

I. Patterns, Relationships and Functions

1. Students recognize similarities and generalize patterns; use patterns to create models and make predictions; describe the nature of patterns and relationships; and construct representations of mathematical relationships. (Patterns)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
5. Apply their experiences with patterns to help solve problems and explore new content.	• developing and using the strategy "look for a pattern." • using numerical pattern relationships as the basis for developing number concepts. Dot patterns  and ten frames  establish visual patterns for developing number concepts and learning basic facts using a part-part-whole model. • using the constant function on a calculator to develop the concept of multiple. 4 + = = = What is your answer? • charting when and how long they watch TV and describing any patterns they observe: - why certain viewing times occur frequently or infrequently. - whether they watch TV before or after dinner. • making conjectures and predictions about what would happen if their pattern was continued or altered.

I. Patterns, Relationships and Functions

2. Students describe the relationships among variables; predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change. (Variability and Change)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
1. Recognize change and variability when it occurs in a variety of settings.	• searching for and exploring changes and variability: - in their environment like cause and effect relationships. - people, relationships, and situations as they change over time (e.g., their own growth, weather, seasons, calendar, time, seeds growing, graphing how many basic facts they get correct in one minute). - sequencing events. - charting and analyzing change or variability as part of a science experiment. - discussing why there are variations in different situations (e.g., results from flipping fair coins or rolling fair dice, distribution of birth months, growth rate of plants in a science experiment, color distribution of candy). • describing how one variable changes in relation to another.

I. Patterns, Relationships and Functions

2. Students describe the relationships among variables; predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change. (Variability and Change)

<table>
<tr><th>In the early years, children have the opportunity to...</th><th>By engaging in activities such as...</th></tr>
<tr><td>2. Recognize that change is often predictable, but variable, and that patterns emerge that help describe change.</td><td>• identifying, analyzing and describing change using a table to record and then identify the pattern when playing "What's My Rule" or using an input/output machine.
What's My Rule?
<table>
<tr><td>input</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>10</td></tr>
<tr><td>output</td><td>1</td><td>3</td><td>5</td><td>7</td><td>9</td><td>?</td></tr>
</table>
• identifying and describing predictable changes in a variety of settings:
- describing how a butterfly develops.
- explaining how the seasons change.
- describing what happens as evening approaches.
- creating a timeline of predictable changes in their school day, school year or life.
• answering "what next?" and "what changes?" questions in order to identify and describe patterns of change.</td></tr>
</table>

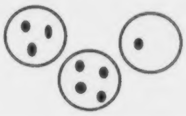
I. Patterns, Relationships and Functions

2. Students describe the relationships among variables; predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change. (Variability and Change)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
3. Explore change and realize that changes are frequently interdependent.	• identifying, describing, modeling and representing how one variable changes in relation to another: - buying more cans of pop increases the cost. - putting more butter on my potato increases the number of calories. - as I get older I get taller. - decreasing the length of the side decreases the number of units in the area. - changing the ratio of one pigment when mixing colors changes the resulting color. • using patterns of change to make predictions, answer questions and solve problems: - developing ratios for mixing colors to get certain pigments. - predicting time of sunset from previously collected data. - solving a problem by looking for a pattern.

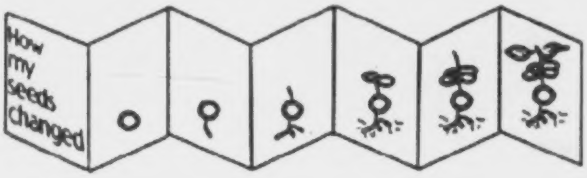
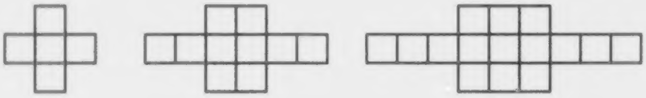
I. Patterns, Relationships and Functions

2. Students describe the relationships among variables; predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change. (Variability and Change)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
4. Use tables, charts, open sentences and hands-on models to represent change and variability.	• exploring and comparing multiple ways to represent change and variability—such as input/output machines, What's My Rule?, mathematical balances, tables, rules/formulas, graphs, open sentences, coordinate graphs. • comparing and relating different representations for change (e.g., formula to table, ordered pairs to coordinate graph). • exploring how the element of chance makes any set of data subject to variation. Put 30 chocolate chips in dough to make 10 cookies. Discuss why each cookie does not have exactly three chips. 

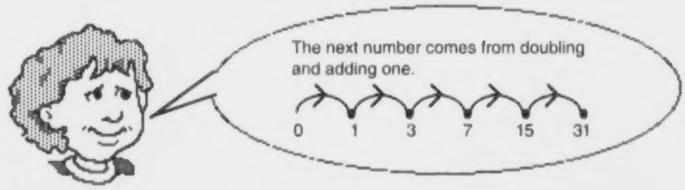
I. Patterns, Relationships and Functions

2. Students describe the relationships among variables; predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change. (Variability and Change)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
5. Begin to describe and differentiate between types of relationships, especially repeating, growing and shrinking patterns.	• exploring different types of relationships and changes like constant, not constant, numerical and spatial patterns (e.g., growing and shrinking). • making a wall story or zigzag book to illustrate a growing story.   What comes next? What is changing?

I. Patterns, Relationships and Functions

2. Students describe the relationships among variables; predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change. (Variability and Change)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
6. Explore variability and change in a variety of contexts, investigations and problems.	• using their knowledge of variability and change to make and defend their conjectures and predictions and to solve problems—such as explaining a prediction for the next term.  The next number comes from doubling and adding one. 0 1 3 7 15 31 Children plant their own amaryllis plant so they can record growth. Sample Activities Related to Variability and Change • Predict a sequence for the growth of the plant. What will happen first? Next? • Make a book illustrating the growth of the plant. • Explore ways to represent the growth of the plant; compare different representations. • Record data on the plant's growth; use the data to predict growth. • Discuss changes that were predictable as well as things that were not predictable. • Compare growth rates of different students' plants, discussing possible reasons for variation in growth. What things will affect how fast our plants are growing? Why might Josh's plant be growing faster than Paul's?

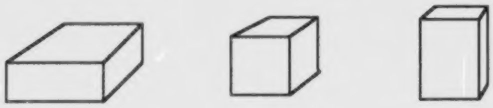
II. Geometry and Measurement

1. Students develop spatial sense, using shape as an analytic and descriptive tool;identify characteristics and define shapes; identify properties and describe relationships among shapes. (Shape and Shape Relationships)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
1. Recognize and name familiar shapes in one, two and three dimensions, such as lines, rectangles and spheres, and discuss the shape of a graph.	• exploring familiar shapes and identifying shapes in their environment such as: - a geometric walk around the school or playground to identify and record the shapes found. - recognize different common shapes in quilts and environmental print (signs, logos, designs). • creating models of two and three dimensional shapes using clay, straws or paper patterns and exploring their uses. Will they roll or can they be stacked? How many faces do you see? • discussing the shape of the graph and describing it using informal language (bumps, clumps, holes, middle).

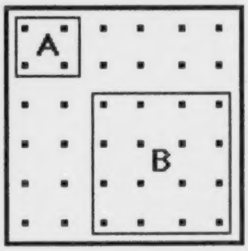
II. Geometry and Measurement

1. Students develop spatial sense, using shape as an analytic and descriptive tool;identify characteristics and define shapes; identify properties and describe relationships among shapes. (Shape and Shape Relationships)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
2. Describe the attributes of familiar shapes.	• describing the attributes of familiar shapes, such as: - a square has four equal sides with square corners (right angles); a cube has six faces with equal sides. - using two-dimensional language (The opposite sides of a rectangle are parallel) and three-dimensional language (The six faces of the cube are congruent) to describe the attributes of various shapes. - using various boxes, describe and compare faces, edges, and vertices.  • finding shapes in the real-world through scavenger hunts such as "Find a red triangle object." • creating or solving logic puzzles to describe familiar two- and three-dimensional shapes.

II. Geometry and Measurement

1. Students develop spatial sense, using shape as an analytic and descriptive tool;identify characteristics and define shapes; identify properties and describe relationships among shapes. (Shape and Shape Relationships)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
3. Compare, sort and classify familiar shapes.	• sorting attribute blocks using a shape attribute (square, triangle). • classifying shapes by an attribute such as: has corners, one curved edge, no curved edge. • comparing two shapes on a geoboard by describing the length of square B as triple the length of square. 

II. Geometry and Measurement

1. Students develop spatial sense, using shape as an analytic and descriptive tool;identify characteristics and define shapes; identify properties and describe relationships among shapes. (Shape and Shape Relationships)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
4. Draw and build familiar shapes.	• using familiar shapes to produce more complex shapes and designs: for example boxes, wood pieces, and cardboard shapes to create a shape city." • constructing a shape on the geoboard and then recording the shape on dot paper. • constructing three-dimensional shapes using marshmallows and tooth picks. • using 4", 6", and 8" straws, to construct as many triangles as possible and then record results in a table.

II. Geometry and Measurement

1. Students develop spatial sense, using shape as an analytic and descriptive tool;identify characteristics and define shapes; identify properties and describe relationships among shapes. (Shape and Shape Relationships)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
5. Explore ways to combine, dissect and transform shapes.	• combining and transforming shapes using Logo to explore creation, transformation, dissection and rotation of shapes. • exploring with tangram pieces to find possible arrangements of pieces as squares • tracing shapes and the image of those shapes after sliding, flipping, and turning them. • combining five square pieces (pentomonoes) to find all possible arrangements. Are any rotated from others? • building an irregular shape on a geoboard and dissecting it into regular shapes to find area. • discovering symmetrical properties by paper folding or reflective devices by: - comparing dissections to find lines of symmetry in nature (a snowflake) or geometric shape. - search through wallpaper books to discover how shapes are tesselated to create various patterns that are repeated. Here's the shape. This is what it looks like flipped, slid, turned. Is this the same shape?

II. Geometry and Measurement

1. Students develop spatial sense, using shape as an analytic and descriptive tool;identify characteristics and define shapes; identify properties and describe relationships among shapes. (Shape and Shape Relationships)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
6. Recognize parallel and perpendicular line segments and figures that have similarity and/or congruence.	• drawing, tracing, or using models to illustrate concepts of parallel and perpendicular lines: - use the edge of a book or paper to recognize straight lines or to show a line parallel or perpendicular to a table. - use a taut string to show parallel edges of a table. - locate parallel lines on lined paper, upper and lower edges of the chalkboard, section lines in the sidewalk. • drawing, tracing, or using models to develop concepts of similarity and congruence: - use tangram pieces to search for congruent and similar shapes. - find examples of similar or congruent shapes in the home (placemats) or other environments (windows in a building floor tiles, sails on sailboats). • comparing car or airplane models as similar to actual object.

II. Geometry and Measurement

1. Students develop spatial sense, using shape as an analytic and descriptive tool;identify characteristics and define shapes; identify properties and describe relationships among shapes. (Shape and Shape Relationships)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
7. Use shape, shape properties and shape relationships to describe the physical world and to solve problems.	• describing the playground equipment in terms of the shape of the equipment. • looking for shapes in advertising brochures, and writing about how the shapes create a pleasing graphic. • describing the characteristics of a three-dimensional object pulled from a box so others can guess or draw the shape on dot paper; use the shape of the data to make conclusions about the probable results and meaning of the data. • describing and solving problems using boxes, cylinders, cubes to make a shape city or robot. •cutting boxes to show what two-dimensional shapes come together to form the original three-dimensional shape.

II. Geometry and Measurement

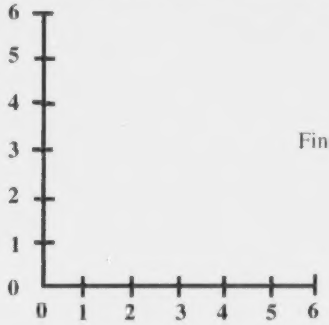
2. Students identify locations of objects, location relative to other objects, and the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object. (Position)

In the early years, children have the opportunity to...	*By engaging in activities such as...*

1. Locate and describe objects in terms of their position, including front, back, inside, outside, right, left, over, under, next to, between, and locations on a number line and on a map.

- giving position words to locate and describe objects like front, back, inside, outside, right, left, over, under, next to, between:
    - draw silly characters or settings: "Draw a tree on the right side of your paper; in the middle of the page draw a funny face; on top of the funny face add a hat; on the left side of the hat add a feather."
    - give position of pegs on the geoboard as inside or outside of the geoband.
    - place a cube on the right side of the desk.
    - use the number line to describe location of a particular number. Number 2 is after (to the right of) the 1 and before (the left of) the 3.

- giving directions:
    - Simon Says: Place your left hand under the table (desk). Place the red cube to the left of the white cube.
    - find a specific point on a map using N-S-E-W to describe the point.
- create patterns or designs on coordinate grids using number pairs.



Find Point A (2,3)

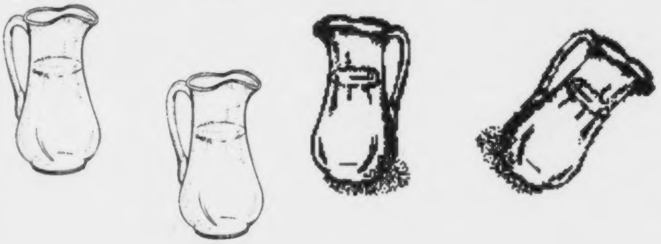
- writing directions to locate a place or object.
    - creating "position" puzzle activities to describe the placement of the tiles.

	B	R
Y	W	G

The B tile is over the W tile.

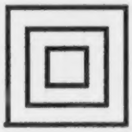
II. Geometry and Measurement

2. Students identify locations of objects, location relative to other objects, and the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object. (Position)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
2. Locate and describe objects in terms of their orientation, direction and relative position including up, down, front, back, N-S-E-W, flipped, turned, translated; recognize symmetrical objects and identify their lines of symmetry.	• locating and describing objects in terms of their orientation: - create a shape; flip, slide or turn the shape; describe the shape in terms of the original shape.  - place a spot of red paint on the paper; fold it and fold again. Where will the spots be? • exploring beginning concepts of symmetry like: - fold paper; cut design along the fold; unfold to find the line of symmetry. - folding paper; placing a small amount of paint on the fold line; refolding; gently pressing the paint towards the edges; opening the design to discover designs with lines of symmetry. - deciding which letters of the alphabet have line or rotational symmetry, which dominos have symmetrical patterns, which leaves have symmetrical design. Explain your decisions. - identifying points from the original point such as in a paint fold where will the red dot occur on the mirror image? • locating and describing objects in terms of directions: - finding a specific point on a map using ordered pairs. - giving directions to a specific spot from a given spot. - locating cities on a map by directional base N-S-E-W. • locating and describing objects in terms of their position: - finding and describing objects that are equidistant to, parallel to, or perpendicular to each other.

II. Geometry and Measurement

2. Students identify locations of objects, location relative to other objects, and the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object. (Position)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
3. Explore what happens to the size, shape and position of an object after sliding, flipping, turning, enlarging, or reducing it.	• using physical objects or computer software programs to explore what happens when objects are flipped, turned or slid: - looking for examples of slides, flips, turns, or enlargement of a pattern in wallpaper and ethnic woven cloth designs. • exploring what happens when an object is enlarged or reduced: - creating scale drawings using grids. - make squares larger by adding borders all around. How many have you added? What would be the next design? 

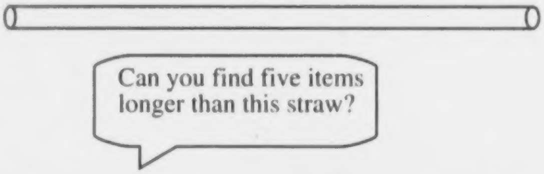
II. Geometry and Measurement

2. Students identify locations of objects, location relative to other objects, and the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object. (Position)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
5. Use concepts of position, direction and orientation to describe the physical world and to solve problems.	• describing situations that require direction, position and orientation concepts: - write the directions for getting home from school using a simplified map of the community. Describe two different possible routes. Which is the most efficient? - give directions for wrapping presents. - list the steps necessary to travel from your classroom to the media center. leave the media center turn left and proceed to the next hallway turn right and proceed to the main office . . . .

II. Geometry and Measurement

3. Students compare attributes of two objects or of one object with a standard (unit) and analyze situations to determine what measurements should be made and to what level of precision. (Measurement)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
1. Compare attributes of objects; develop standard units of measurement; and select and use standard tools for measurement.	• comparing and ordering objects like: - Can you find five items longer (shorter) or the same as your measuring unit? Is this item heavier or lighter than my shoe?  Can you find five items longer than this straw? - weighing each bag: comparing the weights, ordering them from lightest to heaviest (materials in ziplock bags: feathers, cubes, beads,washers, Styrofoam peanuts). - ordering a set of objects from shortest to longest, lightest to heaviest. • using non-standard objects like paper clips, cubes, straws, etc. to measure an object like a table. • developing a need for a standard unit, such as: - using a hand or foot cutout as a non-standard unit; find differences in class results and deciding why a standard unit is needed. - creating their own ruler. • selecting and using standard tools for measurement (rulers, thermometers, scales, graduated cylinders) including both metric and customary units. • arranging wrapped boxes with different weights inside from heaviest to lightest.

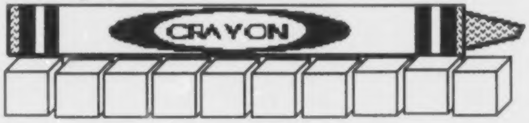
II. Geometry and Measurement

3. Students compare attributes of two objects or of one object with a standard (unit) and analyze situations to determine what measurements should be made and to what level of precision. (Measurement)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
2. Identify the attribute to be measured and select the appropriate unit of measurement for length, mass (weight), area, perimeter, capacity, time, temperature and money.	• identifying the quantity to be measured in a given measuring task and selecting the most appropriate measuring instrument and units of measurement: - decide what is the best way to find the cups of rice in a jar, the weight of pennies in a container, the length of a wall. - decide how we use money. Why do we exchange money for goods? How do we make change? What is meant by saving or spending money? Why would you use coins in some situations, bills in others, and checks in other situations? Creating a story complete with items for purchase, clerks, shop sign and bags heighten student interest in money topics. - use calendars to indicate birthdays of classmates and indicate yesterday, today, tomorrow and next week. - use bus, airline, train schedules, TV schedule, or school bell schedule to explore the idea of elapsed time.

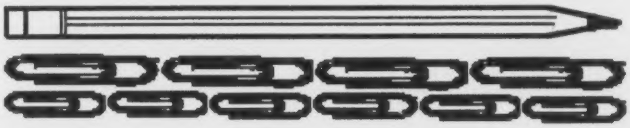
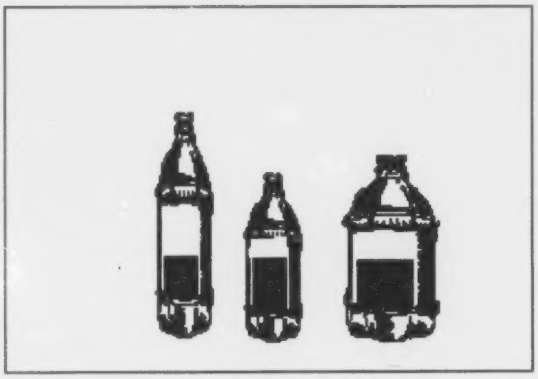
II. Geometry and Measurement

3. Students compare attributes of two objects or of one object with a standard (unit) and analyze situations to determine what measurements should be made and to what level of precision. (Measurement)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
3. Develop strategies for estimating measures and compare the estimates to the results of the measurement; decide if an estimate is "a good estimate."	• estimating measurements for various objects in the environment: - estimate how many scoops of rice are in a container, measure to check if the estimate is reasonable, write and record the actual process involved. - estimate length or width of objects using a block, then record class results on a graph. 

II. Geometry and Measurement

3. Students compare attributes of two objects or of one object with a standard (unit) and analyze situations to determine what measurements should be made and to what level of precision. (Measurement)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
4. Explain the meaning of measurements and recognize that the number of units it takes to measure an object is related to the size of the unit.	• exploring the relationship of one unit to another unit when measuring the same object like: - Why does it take 6 small paper clips but 4 large paper clips to measure the edge of a piece of paper?  • explaining the meaning of the measurement by: - using various containers, judge the amount of liquids in the container. Are they the same amount?  - pouring the same amount of rice or sand into various containers. Why are the heights different? - selecting the correct measuring tool to measure the length of a hallway or the weight of a rock, and stating why that unit was the appropriate unit to use in this situation. - covering a grid with a tracing of their foot and finding out how many units are inside/outside the tracing.

II. Geometry and Measurement

3. Students compare attributes of two objects or of one object with a standard (unit) and analyze situations to determine what measurements should be made and to what level of precision. (Measurement)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
5. Explore scale drawings, models and maps and relate them to measurements of real objects	• exploring scale drawing of real objects: - examining the enlargement of a dinosaur by comparing the drawing to the enlargement by adjusting the scale. - modeling a scale drawing of any room at home or classroom. - enlarging or reducing cartoons using a square grid. - using maps and globes to measure indirectly quantities that are very large. - comparing images from slides of various objects to the actual sizes of the objects.

II. Geometry and Measurement

3. Students compare attributes of two objects or of one object with a standard (unit) and analyze situations to determine what measurements should be made and to what level of precision. (Measurement)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
6. Apply measurement to describe the real world and to solve problems.	• using measurement to describe real-world situations such as: - estimating how much ribbon you need to tie a bow. - designing a label for a bottle or can showing the volume of the container on the label. - deciding the shortest route to school, home, library, and drawing it with the aid of a map. - setting up a bank to solve problems of purchasing materials, recording amounts, credit, debit and the role of money in our society.

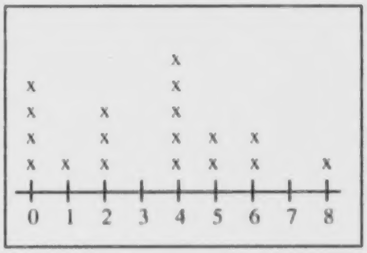
III. Data Analysis and Statistics

1. Students collect and explore data, organize data into a useful form and develop skill in presenting and reading data displayed in different formats. (Collection, Organization and Presentation of Data)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
1. Collect and explore data through counting, measuring, and conducting surveys and experiments.	• suggesting and explaining ways to obtain data. • gathering data in a variety of ways,such as from sorting, counting, measuring and from various sources (e.g., newspaper, magazines, US census). • exploring, designing, and using a variety of strategies to collect data-- such as polls, surveys, experiments, sampling, research, simulations. • raising questions related to their interests and activities which can be answered by collecting, organizing, and presenting data - such as: -How many pokets are you wearing today? -How much taller are fourth graders than first graders? -How much reading do we do at home? -What are our favorite (movies, snacks, colors, cartoon characters?)

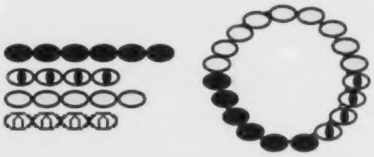
III. Data Analysis and Statistics

1. Students collect and explore data, organize data into a useful form and develop skill in presenting and reading data displayed in different formats. (Collection, Organization and Presentation of Data)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
2. Organize data using concrete objects, pictures, tallies, tables, charts, diagrams and graphs.	• exploring characteristics of data and then sorting and classifying data in order to determine categories—such as discussing various characteristics of shoes before deciding what categories to use for a graph. • investigating different ways to organize and represent data, including opportunities to develop their own strategies before learning standard methods. • organizing and presenting data using different formats—such as: - tallies - tables - object/real graphs - picture graphs - bar graphs - line plots Class Plot for Number of Pockets  - circle graphs - line graphs - coordinate graphing - stem-and-leaf plots - tree diagrams • finding examples of data presentation (e.g., newspaper, baseball card).

III. Data Analysis and Statistics

1. Students collect and explore data, organize data into a useful form and develop skill in presenting and reading data displayed in different formats. (Collection, Organization and Presentation of Data)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
3. Present data using a variety of appropriate representations, and explain the meaning of the data.	• comparing different representations of the same data to develop a sense for how different presentations may provide or highlight different information and to make connections between graphs. For example, colored pop beads could be used to model an object graph of favorite color. The beads could be connected in a necklace to model a circle graph.  • collecting and discussing data displays from print materials, such as newspapers, magazines, and identifying the source of the data. • discussing what the data represent. • displaying information to illustrate both values and categories at the same time—such as graphing heights (value) for different groups of people (categories—first graders, fifth graders, teachers). • making a object graph (graphing the fruits they brought to make a fruit salad) and then recording it by making a picture graph.

III. Data Analysis and Statistics

1. Students collect and explore data, organize data into a useful form and develop skill in presenting and reading data displayed in different formats. (Collection, Organization and Presentation of Data)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
4. Identify what data are needed to answer a particular question or solve a given problem and design and implement strategies to obtain, organize, and present those data.	• identifying what data need to be collected to answer a question or solve a problem, and suggesting strategies for collecting and presenting their data. • exploring and using a process (e.g., understand the problem, gather and explore data, organize and represent the data, describe and interpret the data) for approaching data-analysis problems. • using current data-analysis activities to surface new questions and explorations.

III. Data Analysis and Statistics

1. Students collect and explore data, organize data into a useful form and develop skill in presenting and reading data displayed in different formats. (Collection, Organization and Presentation of Data)

In the early years, children have the opportunity to...	*By engaging in activities such as...*

PORTFOLIO PROJECT

Collecting and saving data-analysis projects, particularly if kept over a number of years, shows growth over time.

Have children pose questions of interest to them which lend themselves to data-analysis.

- How do fourth graders spend after school time?
- Do students in the two second grade classes like the same_______? (e.g., games, foods, sports, TV shows)
- Which movie do we like best: *Aladdin, Beauty and the Beast, Lion King*, or *Cinderella*?
- Do second graders sleep more than fourth graders?

After students select projects, they can discuss strategies for answering their questions, including methods for collecting and displaying data. Projects might include the following steps:

- designing, trying and revising questionnaires, surveys, experiments
- collecting data
- organizing data several different ways
- preparing a presentation for an audience
- writing about their projects
- posing and/or answering related questions

Younger children might need a more structured project such as translating an object graph to a picture/bar graph and then writing statements or answering questions about their graphs.

Projects could be completed and displayed in a class book or in a bulletin board format.

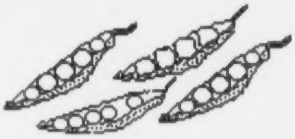
III. Data analysis and Statistics

2. Students examine data and describe characteristics of the distribution; and they relate data to the situation from which they arose and use data to answer questions convincingly and persuasively (Description and Interpretation)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
1. Read and explain data they have collected and organized themselves and progress to reading data from other sources.	• describing and explaining data representations. -a class-created graph representing shoes they are wearing. -a bar graph from the newspaper. • offering descriptive, comparative and interpretative statements about data displays: -What information does the graph provide? - How do ____________ and ______________ compare? - If another class created a graph about ___________, would it look like ours? Why? - What can you tell me about the graph? • providing an appropriate title for a graph. • raising and answering questions about data.

III. Data analysis and Statistics

2. Students examine data and describe characteristics of the distribution; and they relate data to the situation from which they arose and use data to answer questions convincingly and persuasively (Description and Interpretation)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
2. Describe the shape of the data using informal language.	• discussing and describing the visual appearance (shape) of the data using informal language before introducing more formal language—such as spread (range), bumps (mode), clusters, gaps, holes, extreme values (outliers), center (mean, median). "There is a bump at five because most pea pods had five peas in them."  • identifying special features/"landmarks" in data displays—such as spread, middle, or "typical" data point. • exploring and identifying measures of center (mean, median).

III. Data analysis and Statistics

2. Students examine data and describe characteristics of the distribution; and they relate data to the situation from which they arose and use data to answer questions convincingly and persuasively (Description and Interpretation)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
3. Draw, explain and justify conclusions such as trends based on data.	• hypothesizing from a survey of a sample of students to what the results might be if data were collected from all the students. • writing a summary about the results of a survey. • making a presentation of a statistics project that includes answering key questions and exploring conclusions. • using data to defend conclusions and to convince others.

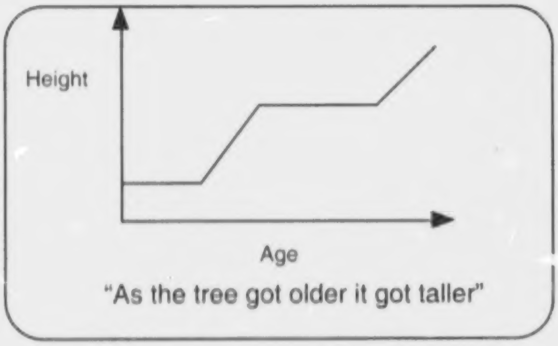
III. Data analysis and Statistics

2. Students examine data and describe characteristics of the distribution; and they relate data to the situation from which they arose and use data to answer questions convincingly and persuasively (Description and Interpretation)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
4. Raise and answer questions about the source, collection, organization and presentation of data as well as conclusions drawn from the data; explore biases in data.	• questioning and discussing the sources for data and the method of collecting data for displays. • learning how different representations of the same set of data can communicate different information about data—such as: - comparing the ways different groups of students displayed the same set of data. - encouraging invented pictures/representations for portraying data sets. - displaying the same data with multiple techniques. • comparing and contrasting different sets of data. • raising new questions based on data displays. • exploring data displays in ways that establish the foundation for understanding bias: - discussing problems they encounter when collecting and displaying data. - comparing different representations for the same set of data. - discussing and evaluating different sets of data. - questioning the source of data and the method of collection. - considering how different representations of the same set of data can be used to communicate different information about the data.

III. Data analysis and Statistics

2. Students examine data and describe characteristics of the distribution; and they relate data to the situation from which they arose and use data to answer questions convincingly and persuasively (Description and Interpretation)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
5. Formulate questions and problems, and gather and interpret data to answer those questions.	 "As the tree got older it got taller" • generating questions and then collecting and interpreting data about topics relevant to them—such as: - interests (favorite food, hobby, book). - current content (science experiment) or events (elections) - school issues/events (class rules, spelling scores).

III. Data analysis and Statistics

2. Students examine data and describe characteristics of the distribution; and they relate data to the situation from which they arose and use data to answer questions convincingly and persuasively (Description and Interpretation)

In the early years, children have the opportunity to...	***By engaging in activities such as...***

PORTFOLIO PROJECT

(continued from Strand III, Standard 1)

In presenting their statistics projects, children could be required to address certain questions or tasks.

Sample Questions and Tasks

- What title would you give the graph?
- What kinds of information can you get from the graph?
- How did the graph help you organize your information?
- What information does the graph provide?
- How was the information collected? What was the source?
- Can you make any predictions based on the information you have?
- Write five statements about your graph.
- What patterns did you find?
- If another class created a graph about the same information, would it look like yours? Why?
- How could this information be used?
- Were there any interesting or unexpected results? Why might this have occurred?
- Describe any problems or question that you had.
- Write some questions which could be answered using your graph.

Variations:

- Have children create a summary page which answers key questions about their data.
- Videotape short "commercials" of students describing and interpreting their data, making the tapes available to parents/guardians.
- As an additional part of the project have children collect graphs from other sources or use other children's graphs and answer selected questions.

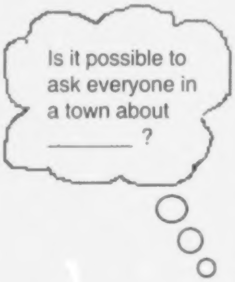
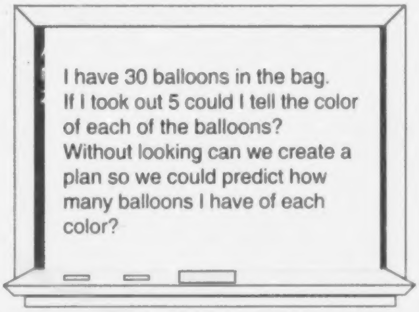
III. Data analysis and Statistics

3. Students draw defensible inferences about unknown outcomes; make predictions and identify the degree of confidence they have in their prediction (Inference and Prediction)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
1. Make and test hypotheses.	• using their data to develop descriptions and theories about their world—such as using the data to make conjectures, to generalize, and to wonder: - How much will we grow this year? - How much are we reading each week? - Are our brothers and sisters older or younger than we are? • exploring how the element of chance makes any set of data subject to variation. • making predictions about outcomes and then comparing predictions with results. - surveying a sample of the class to determine how many pencils they have in their desk. - predicting from the sample to the population (class). - polling the entire class, then comparing results.

III. Data analysis and Statistics

3. Students draw defensible inferences about unknown outcomes; make predictions and identify the degree of confidence they have in their prediction (Inference and Prediction)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
2. Conduct surveys, samplings and experiments to solve problems and answer questions of interest to them.	• designing and conducting surveys, experiments, and investigations; using an investigative approach. - In what month do the most birthdays happen? - How many pockets do students have? - What is the most popular (or common) pet? - How does April weather compare with May weather? - How do students travel to school? • exploring sampling as a way to make inferences about the nature of a population. - investigating the nature of a sample - developing and implementing a sampling plan - comparing results of repeated samples from the same population - making inferences about the population from the results of sampling  Is it possible to ask everyone in a town about _______ ?  I have 30 balloons in the bag. If I took out 5 could I tell the color of each of the balloons? Without looking can we create a plan so we could predict how many balloons I have of each color? • exploring the concept of randomness such as: - drawing students' names at random and collecting data to see how often each student's name is drawn. - discussing strategies for making sure each item has the same chance of being selected.

III. Data analysis and Statistics

3. Students draw defensible inferences about unknown outcomes; make predictions and identify the degree of confidence they have in their prediction (Inference and Prediction)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
3. Formulate and communicate arguments and conclusions based on data and evaluate their arguments and those of others.	• explaining how they reach a decision, giving reasons based on their data. • questioning each other about their conclusions. • checking to see if others agree with their conclusions. • trying to persuade others using the results from an experiment.

III. Data analysis and Statistics

3. Students draw defensible inferences about unknown outcomes; make predictions and identify the degree of confidence they have in their prediction (Inference and Prediction)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
4. Make and explain predictions based on data.	• searching for patterns in data so they can identify trends and make predictions. • making and justifying predictions made from analyzing data.

III. Data analysis and Statistics

3. Students draw defensible inferences about unknown outcomes; make predictions and identify the degree of confidence they have in their prediction (Inference and Prediction)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
5. Make predictions to answer questions and solve problems.	• solving data-analysis problems using an investigative approach which encourages: - raising questions and brainstorming. - understanding the problem. - gathering and exploring data. - describing, interpreting and analyzing data. - making inferences and predictions. - making and implementing decisions. - reflecting back. • making inferences from a sample to a population. • using current data-analysis activities to raise new questions and explorations.

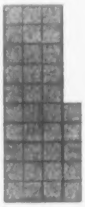
III. Data analysis and Statistics

3. Students draw defensible inferences about unknown outcomes; make predictions and identify the degree of confidence they have in their prediction (Inference and Prediction)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
	SAMPLE DATA-ANALYSIS PROBLEM
Data-analysis problem	The class needs to plan the menu for the all-school picnic.
• raising questions and brainstorming and understanding the problem	- brainstorming ideas about types of food to provide, ways to find out what most people want - questioning and discussing ways to sample the school population - narrowing ideas down and deciding to conduct a survey of a random group - identifying sampling procedures
• gathering and exploring data	- developing, field testing, and revising survey - selecting sample - conducting survey - organizing data in a manner to facilitate decision making
• describing, interpreting, and analyzing data	- explaining the data and what they represents - identifying special features that will facilitate decision making - discussing any problems they had in collecting, organizing or analyzing the data
• making inferences and predictions	- generalizing from the sample to the population
• making and implementing decisions	- deciding on a menu - deciding how much to order - raising new questions, such as whether the menu will work for people who are vegetarians or who have food restrictions.
• reflecting back	- questioning whether something was overlooked. - evaluating results after the picnic

IV. Number Sense and Numeration

1. Students experience counting and measuring activities to develop intuitive sense about numbers; develop understanding about properties of numbers; understand the need for and existence of different sets of numbers; and investigate properties of special numbers. (Concepts and Properties of Numbers)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
1. Develop an understanding of whole numbers; read, write and count using whole numbers; investigate basic concepts of fractions and decimals.	• developing an understanding of whole numbers using: - models that come in pairs like shoes, ears, eyes, mittens, and skip-count by twos; and models that come in triples like tricycles, tripods, and skip-count by threes. - simple nursery songs and finger plays for numbers 1-10. - pictures in newspapers and magazines that show arrays of numbers. • reading, writing, and counting using: - objects to form groups or subsets of a group (shells, rocks, buttons) given a number. - list 20 ways to represent 100 using base ten blocks. Draw and write each response. • investigating initial concepts of fractions and decimals using: - paper folding to create area model of halves, fourths, eighths, sixteenths and drawing equivalent models on graph paper, or length model using colored paper strips folded to represent given fraction. - shade in given amount on decimal mat grids. 

IV. Number Sense and Numeration

1. Students experience counting and measuring activities to develop intuitive sense about numbers; develop understanding about properties of numbers; understand the need for and existence of different sets of numbers; and investigate properties of special numbers. (Concepts and Properties of Numbers)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
2. Investigate and develop an understanding of the base-10 numeration system.	• investigating the base-ten numeration system using: - ten frames and hundreds charts to recognize quantity of number. - the calculator to discover number patterns using the constant feature. • developing place-value concepts, including regrouping: - by forming groups of ten: bundle groups of ten straws, glue beans to tongue depressors, combine objects in ziplock bags or stamp coin values on cards. - using base-ten blocks to present quantity of number, to show trading for regrouping purposes, to show relationship between the place value holder (20 units is the same as 2 longs). - representing a quantity using a place-value holder.

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IV. Number Sense and Numeration

1. Students experience counting and measuring activities to develop intuitive sense about numbers; develop understanding about properties of numbers; understand the need for and existence of different sets of numbers; and investigate properties of special numbers. (Concepts and Properties of Numbers)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
3. Develop an understanding of the properties of numbers (e.g., order) and of the properties of the special numbers 0 and 1.	• developing strategies to learn basic facts like: - turn-around (2 + 3, 3 + 2). - counting-on using 1, 2, 3. - doubles (neighbors, near neighbors) 4 + 4 = 8, then 4 + 5 = 9. • giving examples to illustrate the result of special numbers 0 and 1: - use a hundreds chart or cubes to demonstrate that adding 0 does not change the original value of the number, while adding 1 to the original number gives the next number.

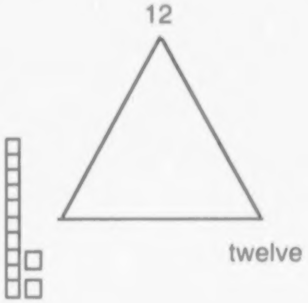
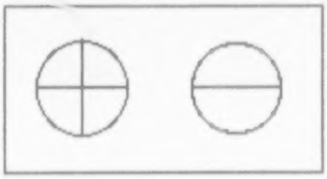
IV. Number Sense and Numeration

1. Students experience counting and measuring activities to develop intuitive sense about numbers; develop understanding about properties of numbers; understand the need for and existence of different sets of numbers; and investigate properties of special numbers. (Concepts and Properties of Numbers)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
4. Apply their understanding of number systems to model and solve problems.	• applying their understanding to solve problem such as: - talking about and recognizing uses of numbers in the environment by reading bottles, boxes, clocks, license plates, meter sticks, money values, phone numbers, road signs, rulers, scales, watches. - working with problems like "If I have 2 bags of candy bars with 14 candy bars in each, will I have enough for everyone in our class? Too many? How many is NOT enough? How many am I short?

IV. Number Sense and Numeration

2. Students recognize that numbers are used in different ways like counting, measuring, estimating and ordering; understand and produce multiple representations of a number; and translate among equivalent representations. (Representation and Uses of Numbers)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
1. Represent whole numbers, fractions and decimals using concrete, pictorial and symbolic representations.	• representing whole numbers, fractions, and decimals in concrete, pictorial and symbolic forms by: - relating sets of objects, oral names, and written symbols to show "how many."  - using flow chart arrows on the calendar for recognizing counting patterns. - forming groups of ten by bundling groups of ten straws, gluing beans to tongue depressors, combining objects in ziplock bags or stamping coin values on cards. - using the ten frame or hundreds chart to display a given quantity. - using fraction pie charts to shade in a given quantity and show relationships between fraction families.  - using decimal mats or graph paper to shade in a given quantity.

IV. Number Sense and Numeration

2. Students recognize that numbers are used in different ways like counting, measuring, estimating and ordering; understand and produce multiple representations of a number; and translate among equivalent representations. (Representation and Uses of Numbers)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
2. Explore and recognize different representations for the same number and explain why they are the same.	• exploring representations of a number by: - sorting a set of objects into different subsets (fact families) and recording answers. - demonstrating how many ways they can represent 35 by using base- ten blocks, counters, money values, and answering "How do you know you have found all possible combinations?" - selecting a number and creating a collage of ways to represent that number such as an array, Roman numeral, number sentences, words.

IV. Number Sense and Numeration

2. Students recognize that numbers are used in different ways like counting, measuring, estimating and ordering; understand and produce multiple representations of a number; and translate among equivalent representations. (Representation and Uses of Numbers)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
3. Investigate ways numbers are used (e.g., counting, ordering, naming, locating, measuring).	• investigating ways numbers are used in: - *counting.* Look in newspapers for large numbers that are rounded and explain what was possible reasoning for rounding those numbers. Look at car ads for the number of cars for sale at a given price. This number counts or tells how many are available. - *ordering.* How are clothing sizes indicated? What does the order progression represent? Why are house numbers assigned in sequential order? - *naming.* How do the keys of a calculator or telephone help us name numbers for accurate answers or correct phone calls? How are numbers used on cars? (license plates, models). - *locating.* How do addresses help you locate a house or restaurant? - *measuring.* What is the relationship between a number and the amount of mulch that is needed for a garden? for building a fence on the perimeter of the property? What do the digits and the placement of the hands on a clock tell us about the current time?

IV. Number Sense and Numeration

2. Students recognize that numbers are used in different ways like counting, measuring, estimating and ordering; understand and produce multiple representations of a number; and translate among equivalent representations. (Representation and Uses of Numbers)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
4. Develop strategies for estimating quantity and evaluate the reasonableness of their estimates.	• using strategies for estimation like front end, rounding, and compatible numbers and then evaluating the reasonableness of the answer through: - estimating the number of items (e.g., pennies, raisins) in a jar. Once estimate is made, remove a set quantity. Estimate again. Continue with this pattern until all groups are removed from the total set.. - drawing a many X's as possible in one minute. First estimate the number of X's. Complete the activity. Use tally marks to find the total. - taking the dinner check and using front end estimation to check the accuracy of the total. - using compatible numbers to solve problems like: There are 17 students and 81 ounces of fruit juice. About how much fruit juice would each child receive?

IV. Number Sense and Numeration

2. Students recognize that numbers are used in different ways like counting, measuring, estimating and ordering; understand and produce multiple representations of a number; and translate among equivalent representations. (Representation and Uses of Numbers)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
5. Select appropriate numbers and representations in order to solve problems.	• working with real-life situations that involve numbers like: - How much ribbon and wrapping paper are needed to wrap five birthday gifts given assorted boxes? - Your family is going on a trip. How much money should you budget? What might be expenses for a day per person? How far can you comfortably travel in a car in a day? What considerations might you take into account (sight-seeing, expressway vs. country roads)? - How many candy bars must your class sell so you can attend a theater performance at the music hall?

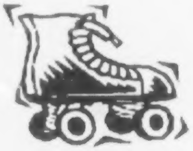
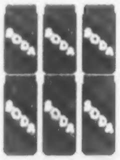
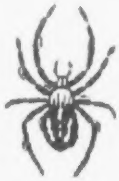
IV. Number Sense and Numeration

3. Students investigate relationships such as equality, inequality, inverses, factors and multiples; and represent and compare very large and very small numbers. (Number Relationships)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
1. Compare and order numbers using "equal," "less than" or "greater than."	• comparing and ordering numbers like: - choosing the smaller or greater number from a pair of cards. - investigating numbers by matching the word with the numeral. - finding one more/one less, or the number before or after a given number using a hundreds grid. - selecting a card from a deck of cards and answering questions like "Is this larger than 10? less than 50?" - presenting an argument that two-thirds is larger than one-half.

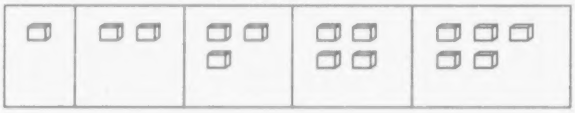
IV. Number Sense and Numeration

3. Students investigate relationships such as equality, inequality, inverses, factors and multiples; and represent and compare very large and very small numbers. (Number Relationships)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
2. Use part-whole relationships to explore numbers, develop number concepts, and understand computation.	• exploring, developing and understanding the part-whole relationship of number using: - collections of materials (blocks, macaroni), separating them into given quantities. Using 7 blocks, separate them into two sets. How many different arrangements can you find? - objects which can represent doubles.    - pairs of wheels on a roller-skate: 2 + 2 = 4. - cans of soda: 3 + 3 = 6. - legs of spiders: 4 + 4 = 8. - representations to create a class booklet. - ten frames to represent the fact families. - fraction strips to explore how many parts make a whole, and what combinations can be used to make equivalent fractions. • exploring different variations of problems using "part-part-whole" like: Ann has 6 green buttons and 8 blue buttons. How many buttons does she have?

IV. Number Sense and Numeration

3. Students investigate relationships such as equality, inequality, inverses, factors and multiples; and represent and compare very large and very small numbers. (Number Relationships)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
3. Classify numbers as even or odd and explore concepts of factors and multiples.	• developing strategies to classify numbers as even or odd by: - using cubes to show that even numbers have a partner while odds are missing a partner.  - using a hundred chart for skip counting; coloring even or odd numbers to find patterns. - measuring their height to see if they are an even or odd number in length. • exploring concepts of factors and multiples using: - a Venn diagram to demonstrate multiples of numbers looking for the common elements. - a hundreds grid, coloring in all multiples of any number and looking for patterns. Compare the different completed number grids looking for common elements. - a calculator to find all the divisors (factors) of 72.

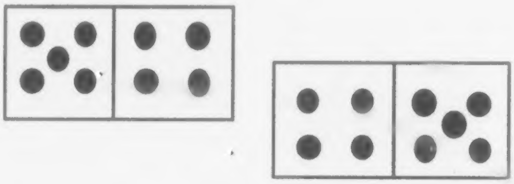
IV. Number Sense and Numeration

3. Students investigate relationships such as equality, inequality, inverses, factors and multiples; and represent and compare very large and very small numbers. (Number Relationships)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
5. Apply their understanding of number relationships in solving problems.	• solving problems that involve real-world applications where understanding of number relationships is demonstrated like: - reading scales such as those found on thermometers, graphs and rulers. - using maps for computing mileage, checking scales to compare population sizes of cities. • engaging in activities using large numbers to understand relationships between millions, billions, trillions as described in *How Much Is A Million*.

V. Numerical and Algebraic Operations and Analytical Thinking

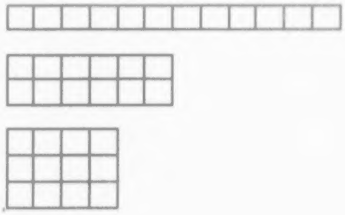
1. Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems. (Operations and their Properties)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
1. Use manipulatives to model operations with numbers; develop their own methods of recording operations; and relate their models and recordings to standard symbolic expressions and algorithms.	• developing personal recording procedures for basic operations before learning traditional recording methods using: - concrete or pictorial representations in number journals. - their recording procedures toexplain basic operations to another person. • modeling operations with numbers using: - dominos to record addition and subtraction sentences. - dominos to find fact families.  - cubes to represent repeated addition patterns ---- + ---- + ---- 4 4 4 3 groups of 4 makes 12 1 2 3 4 -- 5 6 7 8-- 9 10 11 12 • modeling operations with concrete objects, connecting the manipulative model to a symbolic/recorded action.

(CONTINUED)

V. Numerical and Algebraic Operations and Analytical Thinking

1. Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems. (Operations and their Properties)

<table>
<tr><th>In the early years, children have the opportunity to...</th><th>By engaging in activities such as...</th></tr>
<tr><td>(continued from previous page)
1. Use manipulatives to model operations with numbers; develop their own methods of recording operations; and relate their models and recordings to standard symbolic expressions and algorithms.</td><td>• relating their models to standard expressions and algorithms using:
- interlocking cubes as a model for division concepts. Create a train of 14 cubes. Break off groups of twos. How many break-offs are there? Are there any left overs? Repeat with break-offs of threes. Continue the pattern. Record findings on a chart.
<table>
<tr><th>GROUPS OF</th><th>BREAK-OFFS</th><th>LEFTOVERS</th></tr>
<tr><td>2</td><td>7</td><td>0</td></tr>
<tr><td>3</td><td>4</td><td>2</td></tr>
<tr><td>4</td><td>3</td><td>2</td></tr>
<tr><td>5</td><td>2</td><td>4</td></tr>
</table>
- as many rectangles made from 12 tiles as possible. Write the number sentences to describe the rectangles.

</td></tr>
</table>

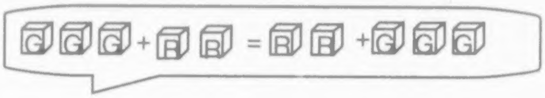
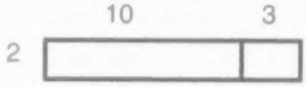
V. Numerical and Algebraic Operations and Analytical Thinking

1. Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems. (Operations and their Properties)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
2. Develop and apply the appropriate method of computation from among mental computation, estimation, paper-and-pencil or calculators; explain why they are choosing a method and how they know which operations to perform in a given situation.	• developing and applying the appropriate method of computation from mental mathematics, estimation, paper/pencil, or using a calculator such as: - "I have 38 toys. I gave 10 to a friend. How would I solve this?" (mental mathematics, calculator, computation) - "I have $24.68. I bought a toy for $2.16, a book for $3.49, and a candy bar for $.69. How much do I have left? Can I buy a T-shirt for $8.94?" Would you solve by mental mathematics, calculator, computation, or estimated answer? • explaining their reasoning for the selection of method and thinking like: What is it that I am supposed to do? - signaling with "thumbs up" or "thumbs down" to indicate their answers to questions like 763 + 402 more (or less) than 1000? Is 430-298 more (or less) than 500? Why? - writing an explanation of their reasoning. • knowing which operation to perform in a given situation. - using *The Doorbell Rang* to explore division strategies. - thinking about and solving multi-step problems.

V. Numerical and Algebraic Operations and Analytical Thinking

1. Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems. (Operations and their Properties)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
3. Explore properties of operations (e.g., commutative and distributive properties) and give examples of how they use those properties.	• exploring properties of operations using: - 3 green and 2 red cubes to demonstrate the commutative property.  - number tiles to show that with any three numbers four facts are generated. 3 + 4 = 7 7 - 4 = 3 4 + 3 = 7 7 - 3 = 4 - base ten blocks to represent the distributive property area = 2 x 10 + 2 x 3 = 2 x (10 + 3) 

V. Numerical and Algebraic Operations and Analytical Thinking

1. Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems. (Operations and their Properties)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
4. Apply operations efficiently and accurately in solving problems.	• solving computational problems which require analyzing the situation, determining operation required, selecting appropriate numbers, computing and evaluating results so a decision can be made like: - If one bicycle has 2 wheels, how many wheels would 3 bikes have? How could you demonstrate this to me? What is the written equation (number sentence)? What if there were 10 bikes? 6 bikes? Draw and write your response. - Can you decorate your room the way you want for $500? - Can you plan a party for eight people for $25?

V. Numerical and Algebraic Operations and Analytical Thinking

2. Students analyze problems to determine an appropriate process for solution and they use algebraic notations to model or represent problems. (Algebraic and Analytic Thinking)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
1. Write and solve open sentences (e.g. ◆ + ▲ = 5) and write stories to fit the open sentence.	• modeling different meanings/uses for variable like: - as a name: My age, or the temperature of the water. - as a changing quantity: If my number doubles, then doubles again, what do I have? Checking the temperature on a daily basis at the same time. - as a place holder: Using the memory key on a calculator. Tell me a story about _ x _ = 12.

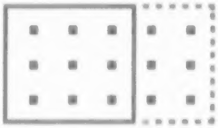
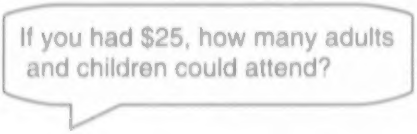
V. Numerical and Algebraic Operations and Analytical Thinking

2. Students analyze problems to determine an appropriate process for solution and they use algebraic notations to model or represent problems. (Algebraic and Analytic Thinking)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
2. Explore algebraic concepts with manipulatives such as balance scales, tables of input and output, and pictorial representations of problems.	• creating pictorial representations of problems like: - pencils must be packed in packages of any combinations of 8 or 6 for a total of 30 in each package. Draw possible packaging. • constructing tables of input and output like: - "Guess My Rule" 5, 7, 9, 11... What's my rule? 20, 16, 12 ... What's my rule? • using a balance scale to write algebraic sentences requiring equivalent weights like: - **m** large paper clips weigh the same as **y** small paper clips.

V. Numerical and Algebraic Operations and Analytical Thinking

2. Students analyze problems to determine an appropriate process for solution and they use algebraic notations to model or represent problems. (Algebraic and Analytic Thinking)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
3. Find replacements for variables in open sentences.	• demonstrating replacements for open sentences like: - providing the missing number given a dot pattern and a card with the total quantity needed. Fill in the missing element. Total Quantity Needed: 15 9 + _ = 15  So 9 + 6 = 15 - exploring the use of rate like determining the number of adults and children who can attend a baseball game if you know the costs of the tickets.  If you had $25, how many adults and children could attend?

V. Numerical and Algebraic Operations and Analytical Thinking

2. Students analyze problems to determine an appropriate process for solution and they use algebraic notations to model or represent problems. (Algebraic and Analytic Thinking)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
4. Use analytic thinking to describe situations and solve problems.	• exploring situations which require analytic thinking like: - working with a calculator to discover the rule. "Work with a partner. First person puts in a number and passes the calculator to the partner who enters a rule (+7), presses =, and returns the calculator to the partner to see if the rule is known." - creating procedures to complete a task. - exploring functions and mathematical relationships through examples such as: What happens when the number of people in a room doubles, doubles again, etc. - exploring ways quantities vary and change depending on constraints such as: How many ways can you build a rectangular pen using 30 meters of fencing?

VI. Probability and Discrete Mathematics

1. Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available; and, they make critical judgments about claims that are made in probabilistic situations. (Probability)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
1. Explain the difference between chance and certainty and give examples to illustrate their understanding.	• exploring and discussing everyday experiences (e.g., weather, sports) and games which involve chance. • creating cards which can be sorted into one of two piles, certain to occur or uncertain. • conducting experiments with simple devices such as coins, spinners or dice; comparing different children's results; and discussing reasons for variation.

VI. Probability and Discrete Mathematics

1. Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available; and, they make critical judgments about claims that are made in probabilistic situations. (Probability)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
2. Compare events and describe them as "more likely" or "less likely" and use the language of fractions to describe simple probabilities.	• informally exploring the concepts of chance and certainty/uncertainty and developing the language of probability—such as more/less likely, equally/not equally likely, certain/uncertain, possible/probable, fair/not fair.

For a week, children listen for, discuss, and record events according to the likelihood they will occur.

EVENT	MORE/LESS LIKELY?	WHY?
We will have outdoor recess.	Less	snowing
School will get out early.	Less	not enough snow

• exploring ways to model the probability of an event occurring:

What is the Probability?

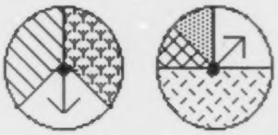
Absolute	Good	Poor	No Way

- of it snowing in July?
- of it raining today?
- of someone losing a tooth this week?

> Open a roll of candy. Look at the colors.
> Assign a probabiity for predicting colors.
> If you opened another roll, what color do you think you would get first? Why?

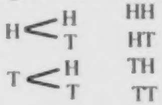
• determining/predicting the likelihood of events and using fractions as a way to describe simple probabilities:
- predicting ratios.
- discussing reasons for predictions.
- exploring part-whole ratios.

• exploring numerical probabilities of outcomes in a variety of situations, with and without equally likely outcomes.



VI. Probability and Discrete Mathematics

1. Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available; and, they make critical judgments about claims that are made in probabilistic situations. (Probability)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
3. Conduct experiments with concrete objects to explore concepts and develop an intuitive under-standing of how the conditions of the experiment can affect the outcome.	• exploring counting problems and experiments which involve recording outcomes of events. • developing strategies for recording outcomes of events—such as pictures and tree diagrams. Tree Diagram for a Two Coin Flip  • conducting simple probability experiments where they can discuss possibilities, make predictions, experiment, and then compare results with the expected outcome.

VI. Probability and Discrete Mathematics

1. Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available; and, they make critical judgments about claims that are made in probabilistic situations. (Probability)

<table>
<tr><th>In the early years, children have the opportunity to...</th><th>By engaging in activities such as...</th></tr>
<tr><td>4. Conduct experiments, record the outcomes, examine those outcomes to determine if they make sense, and search for explanations of the outcomes.</td><td>
• conduct probability experiments which draw on their experiences and interests:
- drawing counters/objects from a bag to explore chance of particular combinations occurring.
- game situations that use spinners and dice.
- tallying outcomes for rolling two dice and finding sum.
<table>
<tr><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td></tr>
<tr><td></td><td></td><td>2+2</td><td>3+2</td><td>3+3</td><td>1+6
4+3</td><td>4+4
6+2</td><td>3+6</td><td></td><td></td><td></td></tr>
</table>
• using two-spinner activities to introduce combining outcomes.

• using an investigative approach to probability which engages them in:
- recording and studying possible outcomes.
- examining results to see if they make sense.
- searching for reasonable explanations for outcomes.
- exploring probability as a ratio or fraction.
-e xploring how conditions affect the outcome.
- comparing experimental with theoretical probability.

• experimenting with methods which generate random outcomes in order to develop a feel for randomness—such as printing student names on tongue depressors and drawing names at random to respond to questions.

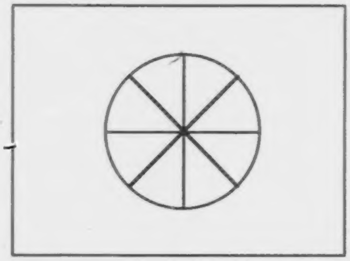
• looking at a spinner or tossing a coin, and telling whether the situation seems fair or unfair, whether outcomes are equally likely or whether outcomes should, or should not, occur an equal number of times.

• discussing situations where results do not "come out right" and exploring:
- how the element of chance makes any set of data subject to variation.
- how data may validate or challenge expectations.
- empirical vs. theoretical probability.
</td></tr>
</table>

(Continued)

VI. Probability and Discrete Mathematics

1. Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available; and, they make critical judgments about claims that are made in probabilistic situations. (Probability)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
(continued from previous page) 4. Conduct experiments, record the outcomes, examine those outcomes to determine if they make sense, and search for explanations of the outcomes.	• making spinners that model certain probabilities.  Color the spinner to model randomly selecting a ball from a bag containing three red, four blue, and one yellow tile. Explain why the spinner models the activity. Compare and discuss results from using the spinner and actually selecting tiles from the bag.

VI. Probability and Discrete Mathematics

1. Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available; and, they make critical judgments about claims that are made in probabilistic situations. (Probability)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
5. Conduct probability experiments and simulations to model and solve problems.	• exploring simulations (i.e., experiments which model probability of real-life situations) that include: - identifying key components and assumptions of the problem. - selecting a random device for key components. - defining a trial. - conducting a large number of trials and recording information. - using data to draw conclusions. - presenting and explaining results. • investigating the likelihood of an event and using data as the basis for making probability statements — such as predicting what types of books children most frequently check out of the library (e.g., by fiction/nonfiction, by general subjects). • exploring how a gumball machine works to investigate probability and model sample space with and without replacement: - What looks most likely? - Is it likely you'll get a red? - Do my chances change? (explore with and without replacement) How? Why? - I got two red in a row. Do you think I'll get a red this time too? Why or why not? • taking samples and using the samples to make predictions.

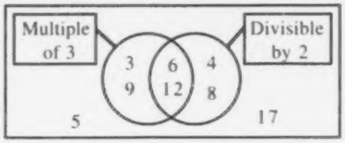
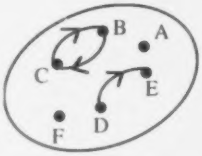
VI. Probability and Discrete Mathematics

2. Students investigate practical situations such as scheduling routing sequencing, networking organizing and classifying; and analyze ideas like recurrence relations, induction and algorithm design. (Discrete Mathematics)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
1. Use manipulatives and diagrams to explore problems involving counting and arranging objects.	• exploring a variety of problems which involve counting and arranging objects and which: - provide practice with simple sequences. - cultivate accuracy. - develop intuition about recurrence methods (i.e., describing future results by looking at previous steps). • developing counting techniques (e.g., pictures, tree diagram) and the fundamental theorem of counting. If one task can be done in *(a)* ways and a second task can be done in *(b)* ways, then the number of ways to perform the two tasks is *(ab)*. e.g. If I take four tops and three pairs of pants on vacation, how many different combinations can I make? • exploring simple permutation, where the order of objects is important, and combinations, where order is not important. Official Ballot for class representatives Vote for two ____Michele ____Toshiba ____Miguel ____Jimmy How many ways can you vote?

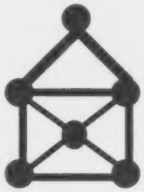
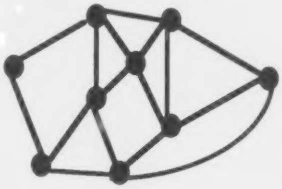
VI. Probability and Discrete Mathematics

2. Students investigate practical situations such as scheduling routing sequencing, networking organizing and classifying; and analyze ideas like recurrence relations, induction and algorithm design. (Discrete Mathematics)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
2. Explore sets and set relationships by sorting and classifying objects.	 Multiple of 3 Divisible by 2 3 9 6 12 4 8 5 17 • sorting and classifying objects by their attributes as a means to explore sets and set relationships. • using diagrams (arrows, Venn diagrams) to represent relationships. If ↗ represents the relation "is the sister of" then which are girls? boys? How many of each are there? How do you know? Are there any whose gender you cannot determine? Which? 

VI. Probability and Discrete Mathematics

2. Students investigate practical situations such as scheduling routing sequencing, networking organizing and classifying; and analyze ideas like recurrence relations, induction and algorithm design. (Discrete Mathematics)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
3. Explore situations in which they model and trace paths using figures consisting of vertices connected by edges.	• informally exploring situations (e.g., networks, relationships, routes, circuits) which can be modeled using vertices connected by edges. Can you trace the figure without lifting your pencil and without retracing any line?  • connecting vertex-edge graphs to familiar experiences such as planning trips or shortest paths, planning bus routes. THE FIRE HYDRANT PROBLEM: The lines represent streets. The dots represent corners. Fire hydrants need to be placed at corners so that each street can get water in case of a fire. Where can fire hydrants be placed to serve all streets using the fewest hydrants? 

VI. Probability and Discrete Mathematics

2. Students investigate practical situations such as scheduling routing sequencing, networking organizing and classifying; and analyze ideas like recurrence relations, induction and algorithm design. (Discrete Mathematics)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
4. Explore now-next patterns.	• exploring pattern activities which repeat a procedure over and over to develop a sequence (iteration): - making a one-difference train using attribute materials. - continuing numerical patterns. 2, 7, 12, 17, 22, 27, ☐, ☐, ☐ (What comes next? Explain.) - skip counting. - repeating (iterating) a unit to measure the length of an object. • exploring pattern activities which involve trying to describe what comes next by looking at previous steps (recursion): - using LOGO programs to create designs. - exploring ways recursive number patterns develop. -extending the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, ...

VI. Probability and Discrete Mathematics

2. Students investigate practical situations such as scheduling routing sequencing, networking organizing and classifying; and analyze ideas like recurrence relations, induction and algorithm design. (Discrete Mathematics)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
5. Explore, develop and invent their own algorithms to accomplish a task or to solve numerical problems.	• sequencing events/steps: -arranging and explaining a sequence of pictures for baking a cake. -exploring systematic approaches (i.e., algorithmic thinking) for completing a task. -developing their own algorithms/directions/sequences for completing tasks, both numerical and non numerical. Write a recipe for making your favorite cookies. • reassembling short stories/nursery rhymes/cartoons that have been cut apart. • creating and solving logic riddles for different materials such as coins, attribute blocks and geometric shapes. I have 6 coins. One coin is used more than once. No nickels are used. • constructing and comparing their own recording procedures for the basic operations (e.g., multiplication, division) instead of/before learning traditional methods for recording the operations.

VI. Probability and Discrete Mathematics

2. Students investigate practical situations such as scheduling routing sequencing, networking organizing and classifying; and analyze ideas like recurrence relations, induction and algorithm design. (Discrete Mathematics)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
6. Use discrete mathematics concepts as described above to model situations and solve problems; and look for whether or not there is a solution (existence problems), determine how many solutions there are (counting problems), and decide upon a best solution (optimization problems).	• looking for multiple solutions to a problem. • discussing whether there is a best solution. - dividing objects (e.g., land, possessions, food, tasks) of different values equitably. - planning efficient routes. • justifying their thinking as a way to help clarify their reasoning -- such as asking questions like: Why? How do you know? What makes you think that? - organizing a complex task and sequencing events so all components get done on time. • coloring maps/drawings with fewest colors so regions sharing boundaries do not use the same color (minimize conflicts).

I. Patterns, Relationships and Functions

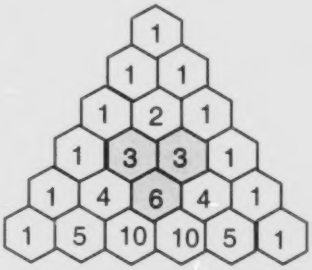
1. Students recognize similarities and generalize patterns; use patterns to create models and make predictions; describe the nature of patterns and relationships,and construct representations of mathematical relationships. (Patterns)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
1. Describe, analyze and generalize patterns arising in a variety of contexts and express them in general terms.	• recognizing patterns arising from everyday situations (e.g., geometric patterns in a quilt, symmetry patterns in art and nature, numerical patterns in a calendar) and describing them verbally and symbolically.

June						
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

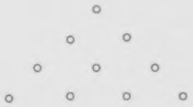
• exploring numeric, algebraic and geometric examples of patterns such as patterns of repeating decimals (numeric), patterns in Pascal's triangle (algebraic), or patterns of change in the area of a square as the length of the side changes (geometric).

1/7 = .142857142857...
2/7 = .285714285714...
3/7 = .428571428571...
4/7 = ???
.
.
.



extending patterns by adding the next few elements or by supplying missing elements.

$T_1 = 1, T_2 = 3, T_3 = 6, T_4 = 10 \ldots T_9 = ?$



• creating and describing original patterns; modeling patterns using physical objects such as counters, attribute pieces, cubes, etc.; and providing reasons to explain their choices when creating or extending patterns.

(continued)

I. Patterns, Relationships and Functions

1. Students recognize similarities and generalize patterns; use patterns to create models and make predictions; describe the nature of patterns and relationships,and construct representations of mathematical relationships. (Patterns)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*	
(continued from previous page) 1. Describe, analyze and generalize patterns arising in a variety of contexts and express them in general terms.	• generalizing a rule to describe a pattern, for example a rule for forming the next Fibonacci number or a rule for determining the number of degrees in the angles of a polygon.	$S_1 = 1$ $S_2 = 4 = 1 + 3$ $S_3 = 9 = 1 + 3 + 5$ $S_4 = 16 = 1 + 3 + 5 + 7$ $S_{10} = ?$

I. Patterns, Relationships and Functions

1. Students recognize similarities and generalize patterns; use patterns to create models and make predictions; describe the nature of patterns and relationships,and construct representations of mathematical relationships. (Patterns)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
2. Represent and record patterns in a variety of ways including tables, charts and graphs, and translate between various representations.	• using a table to show the number of handshakes when every person in the room shakes hands with everyone else. • using a graph to show the total distance traveled when moving at a fixed rate of speed. • writing a mathematical expression to describe the total receipts after selling both adults' and children's tickets. • drawing a graph to represent a mathematical equation. • writing an equation to describe a graph. • matching a graph to a table of values. • organizing data in tables in ways that highlight underlying patterns.

I. Patterns, Relationships and Functions

1. Students recognize similarities and generalize patterns; use patterns to create models and make predictions; describe the nature of patterns and relationships,and construct representations of mathematical relationships. (Patterns)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
3. Use patterns and their generalizations to make and justify inferences and predictions.	• analyzing patterns and using them to predict unknown outcomes, for example the number of diagonals in an n-sided polygon or the tenth term in a sequence of numbers.

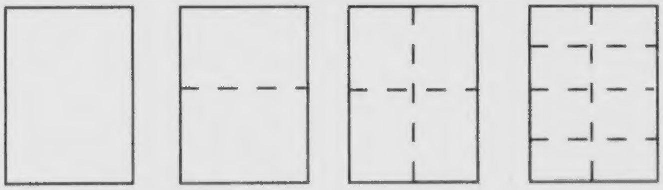
Tower of Hanoi: Given three posts and disks of varying diameter, how many moves are needed to transfer n disks from one post to another? Only one disk may be moved at a time, and larger disks may not be placed on smaller disks.

Tower of Hanoi

Number of Discs	Number of Moves
1	1
2	3
3	7
4	15
•	•
•	•
•	•

- testing and evaluating conjectures based on observed patterns.
- using patterns to solve problems, such as comparing total earnings over time to determine which of two wage plans is most beneficial.

Pat is going to make name tags for the science fair participants, and she has been experimenting to see how many name tags she can cut from a sheet of paper. How many will she be able to get if she continues folding the paper as shown? What is happening to the size of the name tags as she continues to fold? What is a reasonable number of name tags to make from a standard sheet of paper? Explain your recommendation.



I. Patterns, Relationships and Functions

1. Students recognize similarities and generalize patterns; use patterns to create models and make predictions; describe the nature of patterns and relationships,and construct representations of mathematical relationships. (Patterns)

In the middle years, students have the opportunity to...

4. Explore and describe visual and numeric patterns, including linear expressions, near-linear patterns, and symmetric and spatial patterns.

By engaging in activities such as...

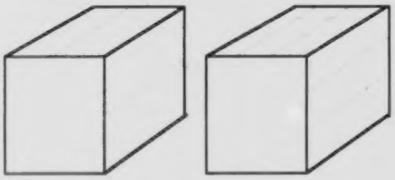
- generating data that exhibit linear patterns, for example by investigating the relationship between diameters and circumferences of circles or the relationship between the cost of a long distance telephone call and the length of the call.

Phone Rate:

Minutes	Cost
1	$0.70
2	$0.85
3	$1.00
4	$1.15
5	$1.30

- comparing patterns and classifying them as linear (e.g., the temperature of a liquid being warmed at a steady rate), near linear (e.g., the cost of postage vs. the weight of the letter), or not linear (e.g., the area of a square vs. the length of its side, or the number of cells present in a culture if the cells divide every 24 hours).

- finding or creating examples to illustrate varius types of patterns.



Colored Faces of Unit Cubes

	#	6	3	2	1	0
Number of Cubes on an Edge	1	1	-	-	-	-
	2	-	8	-	-	-
	3	-	8	12	6	1
	4	-	8	24	24	8
	5	-	8	36	54	27
	6	-	?	?	?	?

I. Patterns, Relationships and Functions

1. Students recognize similarities and generalize patterns; use patterns to create models and make predictions; describe the nature of patterns and relationships,and construct representations of mathematical relationships. (Patterns)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
5. Use patterns and generalizations to solve problems and explore new content.	• identifying and describing patterns in open-ended situations, such as the patterns found in a multiplication table or in Pascal's triangle. . • exploring situations in which the heuristics of looking for patterns and making tables, charts and graphs lead to the solutions of interesting problems.

Locker problem: 1000 lockers, 1000 students …

S1 opens every locker
S2 closes (changes) every second locker
S3 changes every third locker
S4 changes every fourth locker
… and so on.

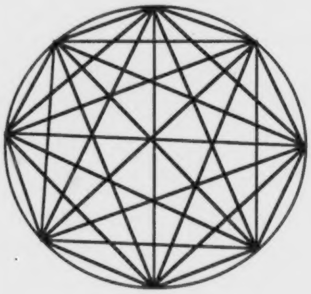
Which lockers will be open in the end?

STUDENT #	LOCKER #													
#	1	2	3	4	5	6	7	8	9	10	11	12	13	14
12												X		
11											X			
10										X				
9									O					
8								X						
7							X							O
6						X						O		
5					X					O				
4				O				O				X		
3			X			O			X			O		
2		X		X		X		X		X		X		X
1	O	O	O	O	O	O	O	O	O	O	O	O	O	O

(continued)

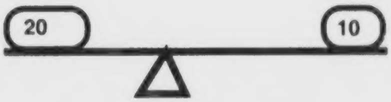
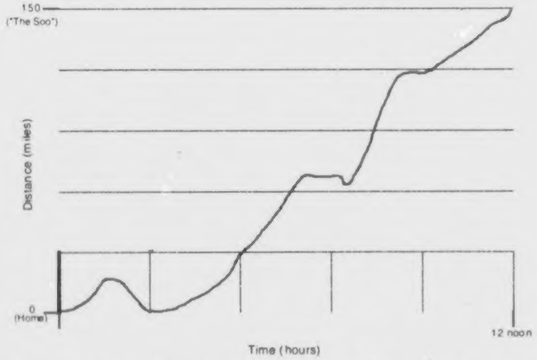
I. Patterns, Relationships and Functions

1. Students recognize similarities and generalize patterns; use patterns to create models and make predictions; describe the nature of patterns and relationships,and construct representations of mathematical relationships. (Patterns)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
(continued from previous page) 5. Use patterns and generalizations to solve problems and explore new content.	• describing the connections among patterns observed in various contexts, for example, recognizing the triangular numbers when they occur in seemingly dissimilar situations.  How many handshakes occur if each of the 8 people in a room shakes hands with every other person? How are these two situations related to triangular numbers?

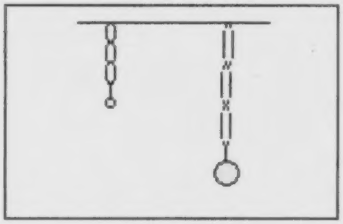
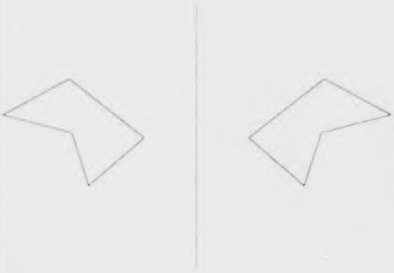
I. Patterns, Relationships and Functions

2. Students describe the relationships among variables; predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change. (Variability and Change)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
1. Identify and describe the nature of change; recognize change in more abstract and complex situations and explore different kinds of change and patterns of variation.	• exploring mathematical patterns and physical situations that lead them to encounter variables in a natural way—such as the effect that varying the amount of light or water has on the growth of a seedling—and describing how one variable changes in relation to another. • charting and graphing the growth patterns of several seedlings and observing the natural variation in height even when the plants have been exposed to identical growing conditions. • conducting experiments in which they systematically manipulate variables, such as using a balance beam to study relationships between weights and their distances from the fulcrum or projecting shadows on a screen to study relationships between shadow size and the object's distance from the screen.  • examining graphs, such as a graph of distance vs. time, and explaining the changes that the graph represents. Trip to Sault Ste. Marie 

I. Patterns, Relationships and Functions

2. Students describe the relationships among variables; predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change. (Variability and Change)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
2. Connect an initial state to a final state and generalize a rule that describes a pattern of change.	• conducting experiments and observations and generating data to explore relationships among dependent and independent variables in diverse contexts, for example by measuring the length of a spring or rubber band as various weights are suspended from it.  • describing what changes and what remains unchanged during explorations such as transforming geometric shapes using slides, flips, and turns, or adjusting the class's test scores by adding 12 points to each person's score.  The polygon on the right has been reflected over the line to form the image on the left. What changed and what remained the same after the reflection? • playing games of "guess the rule" to develop generalizations about patterns of change, giving reasons for their guesses, and expressing the rules verbally and symbolically.

I. Patterns, Relationships and Functions

2. Students describe the relationships among variables; predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change. (Variability and Change)

In the middle years, students have the opportunity to...

By engaging in activities such as...

3. Begin to investigate applications in bivariate data and linear relationships, and explore questions of what will happen to one quantity if another variable is changed.

- using patterns of change to make conjectures and predictions and to answer questions such as:
    - If the observed pattern continues, what will be the value of the variable one year from now?
    - If the observed pattern has been unchanged, what was the value of the variable one month ago?

> Matt shared a secret with his friend, Paul, and made Paul promise not to tell anyone. But the next day, Paul told two people; and the day after that, those two people each told two people. If this pattern continues, how long will it be before the entire school (enrollment 1200) knows the secret?

- conducting experiments to generate data, for example by measuring and comparing the circumferences and diameters of various round objects.

- using technology, especially spreadsheets and calculators, to generate data and explore patterns of variability.

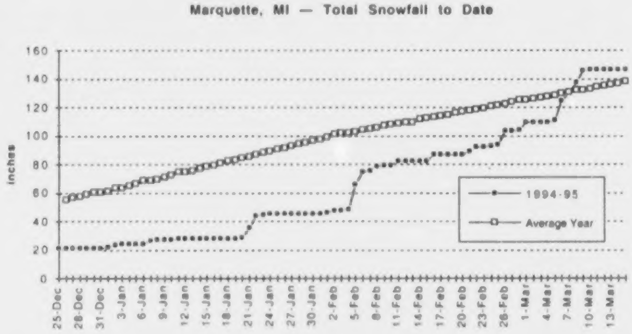
Circle relationships

radius	diameter	area
1	3.14	3.14
2	6.28	12.56
3	9.42	28.26
4	12.56	50.24
5	15.70	78.50
6	18.84	113.04
7	21.98	153.86
8	25.12	200.96
9	28.26	254.34
10	31.40	314.00
11	34.54	379.94
12	37.68	452.16

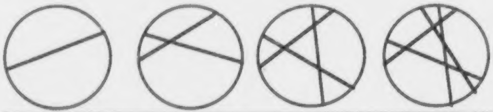
I. Patterns, Relationships and Functions

2. Students describe the relationships among variables; predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change. (Variability and Change)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
4. Represent variability or change by ordered pairs, tables, graphs and equations.	• expressing patterns of change in several different ways, for example by constructing a graph to show the total snowfall accumulation during the winter or by developing a table to show the number of pieces of pizza that result when a pizza is sliced by one, two, three, etc., cuts.



(The graph above shows the weather bureau's reported total-snowfall-to-date in Marquette, Michigan, during the 1994-95 winter as well as the "average" or "normal" seasonal snowfall. One of the graphs appears nearly linear while the other is very irregular. What accounts for this difference in the two graphs? What information is concealed by the graph of average snowfall? In what ways was 1994-95 "average" and in what ways was it not?)



# OF CUTS	0	1	2	3	4	5	6
# OF PIECES	1	2	4	7	11	?	?

• describing plausible situations that could have produced a given graph or pattern.

I. Patterns, Relationships and Functions

2. Students describe the relationships among variables; predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change. (Variability and Change)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
5. Differentiate between functions and relations such as linear vs. not linear or continuous vs. non-continuous.	• exploring and classifying examples of relationships, including: -linear relationships (the relationship between Fahrenheit and Celsius temperatures). -quadratic relationships (the relationship between the area of a square and the length of its side). -inverse relationships (the relationship between speed and time when traveling a fixed distance). -patterns of growth or decay (the number of cells present at the end of the month if the cells divide every 48 hours). -non-continuous functions (the cost of postage to mail letters of varying weight). • classifying observed patterns of change as linear or non-linear. • expressing linear relationships graphically, algebraically and verbally.

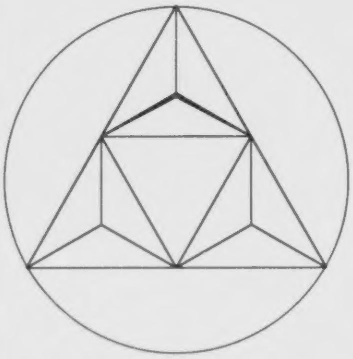
I. Patterns, Relationships and Functions

2. Students describe the relationships among variables; predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability, and compare patterns of change. (Variability and Change)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
6. Continue to explore relationships arising from interesting contexts, and use variables and relationships to solve mathematical problems.	• developing and manipulating physical and mathematical models of real phenomena (for example, the period of a swinging pendulum or the rebound height of a bouncing ball) and describing the associated variables and their relationships. • using a spreadsheet to compare the advantages of investment plans offering different rates of interest.

II. Geometry and Measurement

1. Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes; identify properties and describe relationships among shapes.
(Shape and Shape Relationships)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
1. Distinguish among shapes and differentiate between examples and non-examples of shapes based on their properties; generalize about shapes of graphs and data distributions.	• exploring diagrams and two- and three-dimensional objects in order to: - describe the shapes of the objects. - name common shapes found in the objects. - identify special characteristics and properties of shapes. - classify objects according to their shapes. - differentiate between examples and non-examples of particular shapes. - develop hierarchies of related shapes.  Identify and describe all the shapes that you recognize in the diagram to the left. • going on a geometry walk in the neighborhood and identifying and recording shapes that are observed. • exploring data sets and giving informal descriptions of the data distributions, such as symmetric vs. skewed distributions, positive associations, exponential growth, inverse variations, etc.

II. Geometry and Measurement

1. Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes; identify properties and describe relationships among shapes.
(Shape and Shape Relationships)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
2. Generalize the characteristics of shapes and apply their generalizations to classes of shapes.	• using paper folding, geoboards, and other models to generate and test hypotheses about families of shapes and their properties (e.g., the opposite sides of the rectangles are always equal). • exploring physical models of shapes to determine their symmetries, and describing and demonstrating the symmetries of two- and three-dimensional shapes. • utilizing computer drawing programs to investigate shapes and shape relationships. • drawing or constructing and naming shapes that satisfy given criteria such as "a four-sided figure with one pair of parallel sides."

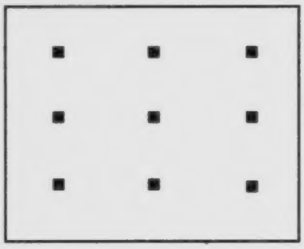
II. Geometry and Measurement

1. Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes; identify properties and describe relationships among shapes.
(Shape and Shape Relationships)

<table>
<tr><th>In the middle years, students have the opportunity to...</th><th>By engaging in activities such as...</th></tr>
<tr><td>3. Derive generalizations about shapes and apply those generalizations to develop classifications of familiar shapes.</td><td>
• developing and verifying generalizations about geometric properties using informal means such as paper folding, reflecting with a mira, or manipulating shapes with a dynamic computer drawing program.
The number of faces, vertices, and edges of a polyhedra is related ($v+f-e=2$)
<table>
<tr><th>Polyhedra</th><th>Vertices</th><th>Faces</th><th>Edges</th></tr>
<tr><td>cube</td><td>8</td><td>6</td><td>12</td></tr>
<tr><td>octahedron</td><td>6</td><td>8</td><td>12</td></tr>
<tr><td>tetrahedron</td><td>4</td><td>4</td><td>6</td></tr>
<tr><td>dodecahedron</td><td>?</td><td>12</td><td>?</td></tr>
<tr><td>icosahedron</td><td>?</td><td>20</td><td>?</td></tr>
</table>
• developing hierarchies for families of shapes, such as drawing a Venn diagram to show the relationships among quadrilaterals, parallelograms, rectangles, squares, rhombi, trapezoids, etc.
• investigating when a characteristic of a shape is sufficient to define a shape, such as determining whether congruent diagonals guarantees that a quadrilateral is a square.
</td></tr>
</table>

II. Geometry and Measurement

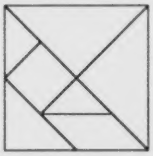
1. Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes; identify properties and describe relationships among shapes.
(Shape and Shape Relationships)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
4. Construct familiar shapes using coordinates or appropriate tools (including technology); sketch and draw two- and three-dimensional shapes.	• constructing shapes using simple materials such as straws, toothpicks, pattern blocks, geostrips, or geoboards.  "How many quadrilaterals can be made on a 3-peg by 3-peg geoboard?" • constructing shapes using various tools including Euclidean tools (compass and straight edge), miras or other reflection devices, computer drawing tools, coordinate graphs, paper folding, and tangram pieces. • constructing shapes that conform to given specifications (e.g., make a trapezoid with two right angles), and determining when it is impossible to create a certain shape (e.g., make a parallelogram with only two right angles).

II. Geometry and Measurement

1. Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes; identify properties and describe relationships among shapes.
(Shape and Shape Relationships)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
5. Combine, dissect and transform shapes.	• combining familiar shapes to produce more complex shapes and designs.

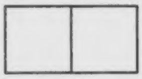


Can you combine all seven tangram pieces to form a rectangle? a parallelogram? a triangle?

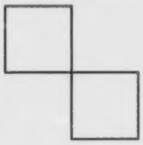
A *domino* is formed when two congruent squares are connected along a side. Similarly, a *triomino* is formed by three connected squares. How many distinct shapes can you find for each of the following?

Name of Shape	Number of Connected Squares	Number of Distinct Arrangements
Domino	2	
Triomino	3	
Tetromino	4	
Pentomino	5	

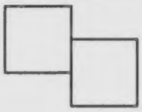
is a domino



is not a domino



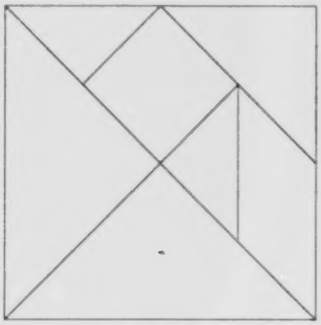
is not a domino



- separating complex shapes into simpler component shapes.
- predicting and investigating the shapes that result when two objects intersect, such as when a plane slices through a cube.
- determining which shapes will tessellate the plane, and creating original tessellations.
- identifying shapes found in artistic designs, especially designs from different cultures.
- using various plane and solid shapes to create original designs or sculptures.
- tracing shapes and images of those shapes after sliding, flipping or turning them.

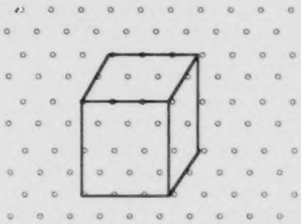
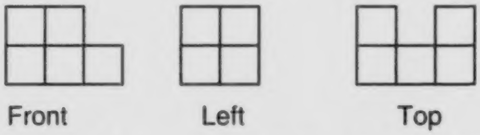
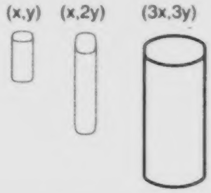
II. Geometry and Measurement

1. Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes; identify properties and describe relationships among shapes.
(Shape and Shape Relationships)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
6. Generalize about the common properties of similar, congruent, parallel and perpendicular shapes and verify their generalizations informally.	• drawing, tracing, folding, and cutting a variety of shapes and using them to develop concepts of *congruent* and *similar* shapes.  Cut the tangram diagram into seven pieces. How do the size and shape of the two small triangles compare to the size and shape of the three larger triangles? Are any of the triangles similar? Are any of the triangles congruent? Give an argument to prove your answer. • exploring physical objects such as globes or models of geometric solids to develop concepts of *perpendicular* and *parallel* lines and planes. • identifying examples of congruence, similarity, parallelism and perpendicularity from the environment and from geometric objects.

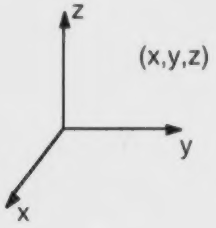
II. Geometry and Measurement

1. Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes; identify properties and describe relationships among shapes.
(Shape and Shape Relationships)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
7. Use shape, shape properties and shape relationships to describe the physical world and to solve problems.	• exploring the connections between shape and numbers by investigating figurate numbers (triangular, square...), primes vs. composite numbers, the shape of data distributions, the shapes of graphs, etc. • using isometric dot paper to draw two-dimensional representations of three-dimensional objects, and constructing three-dimensional models from two-dimensional renderings.  • investigating diagrams of shapes as viewed from different angles and constructing 3-dimensional models to correspond to the 2-dimensional views.  • drawing shapes on a coordinate grid and multiplying the coordinates of selected points by a constant to produce one-way [e.g., (x,y) Æ (x, 2y)] and two-way [e.g., (x,y) Æ (3x, 3y)] distortions.  • conducting open-ended investigations involving shapes, such as coloring maps or finding all the pentominoes and determining which pentominoes can be folded to make an "open box."

II. Geometry and Measurement

2. Students identify locations of objects, location relative to other objects, and the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object. (Position)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
1. Locate and describe objects in terms of their position, including compass directions, Cartesian coordinates, latitude and longitude, and midpoints	• locating objects in relation to other objects (e.g., one meter to the right of the window; two feet above the floor...). • describing positions in terms of compass directions. • reading and following maps. • drawing maps and giving verbal directions for locating objects (e.g., how to get to the library from the school). • specifying Cartesian coordinates for points on a grid, and locating points given their coordinates. • locating and describing points in three-dimensional space using positive (x, y, z) coordinates.  • locating and describing points on a globe using latitude and longitude.

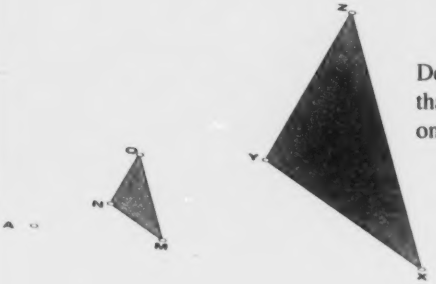
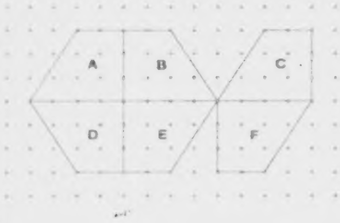
II. Geometry and Measurement

2. Students identify locations of objects, location relative to other objects, and the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object. (Position)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
2. Locate and describe objects in terms of their orientation and relative position, including coincident, collinear, parallel, perpendicular; differentiate between fixed (e.g., N-S-E-W) and relative (e.g., right-left) orientations; recognize and describe examples of bilateral and rotational symmetry.	• locating and describing points, lines, or physical objects in geometric terms such as "on the perpendicular bisector" or "at the intersection". • using physical objects, graphs or sketches to represent positions such as "parallel to," "at the midpoint," or "equidistant from". • using a geometry drawing program to construct objects such as the midpoint of a segment or the perpendicular to a line. • using miras or mirrors to determine whether or not figures or objects have bilateral symmetry. • using tracings and models to determine whether a figure or object has rotational symmetry. • demonstrating that fixed orientations or directions (e.g., two blocks north of my house) yield consistent results, while relative orientations (e.g., go three miles to the right) can result in different outcomes. • examining a set of test scores to determine whether the score distribution is symmetrical.

II. Geometry and Measurement

2. Students identify locations of objects, location relative to other objects, and the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object. (Position)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
3. Describe translations, reflections, rotations and dilations using the language of transformations, and employ transformations to verify congruxnce of figures.	• using physical objects, geometric sketches, and computer drawing tools to explore the effects of translating, reflecting, or rotating a point or object. • drawing, both with paper and pencil and with computer drawing tools, the resulting image when given a description of the transformation of a shape or object (e.g., translate 10 centimeters at an angle of 30° from the horizontal; reflect over the y-axis; rotate 60° about the vertex).  Describe the transformation that projected triangle MNO onto triangle XYZ. • describing the transformation that produced an observed result when shown a drawing of a shape and its image.  Describe one motion that will transform: E to F B to E B to D C to E A to C F to A B to A C to F Describe a combination of two transformations that will transform: A to C F to A D to B D to C

II. Geometry and Measurement

2. Students identify locations of objects, location relative to other objects, and the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object. (Position)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
4. Locate the position of points or objects described by two or more conditions; locate all the points (locus) that satisfy a given condition.	• locating points or objects that satisfy multiple conditions (e.g., on the line and the circle; three blocks from the bank and two blocks from the post office). • writing a set of conditions that allow another student to accurately locate a given object.

II. Geometry and Measurement

2. Students identify locations of objects, location relative to other objects, and the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object. (Position)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
5. Use concepts of position, direction and orientation to describe the physical world and to solve problems.	• engaging in treasure hunts that require following a series of directions to locate a hidden object. • drawing treasure maps that lead to selected hiding places. • giving written or oral instructions to lead another student to a desired location. • viewing objects from different locations and drawing and describing how the perspective changes. • projecting images of shapes and determining how dilations can be used to enlarge or reduce the original shape. • constructing three-dimensional models from two-dimensional drawings, and drawing two-dimensional representations of three-dimensional objects.

II. Geometry and Measurement

3. Students compare attributes of two objects or of one object with a standard (unit) and analyze situations to determine what measurements should be made and to what level of precision. (Measurement)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
1. Select and use appropriate tools; measure objects using standard units in both the metric and common systems, and measure angles in degrees	• measuring everyday objects using: - other objects for comparison (e.g., index cards, cubes, string). - standard units, both common and metric. - common measuring tools (rulers, protractors, thermometers, scales, graduated cylinders, etc.) - rulers that are shorter than the object to be measured (i.e. must be applied more than once) and "broken rulers" whose scales do not begin at zero. • engaging in hands-on activities to develop concepts of measurements in various dimensions, such as using string to surround an object (perimeter), using tiles or paper to cover a surface (area), or using sand to fill a container (volume). • measuring common objects using rulers, trundle wheels, balances, scales, stop watches, clocks, thermometers, graduated cylinders, protractors, and other appropriate measurement tools. • reading and interpreting measuring devices such as meters, scales of various design, calipers, etc.

II. Geometry and Measurement

3. Students compare attributes of two objects or of one object with a standard (unit) and analyze situations to determine what measurements should be made and to what level of precision. (Measurement)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
2. Identify the attribute to be measured and select the appropriate unit of measurement for length, mass (weight), time, temperature, perimeter, area, volume, angle.	• identifying the quantity to be measured in a given measuring task and selecting the most appropriate measuring instrument and units of measurement. • explaining why certain units are preferred over others in a given measurement situation, for example by telling why miles is not a convenient unit for measuring the length of a table.

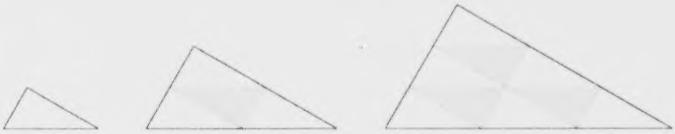
II. Geometry and Measurement

3. Students compare attributes of two objects or of one object with a standard (unit) and analyze situations to determine what measurements should be made and to what level of precision. (Measurement)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
3. Estimate measures with a specified degree of accuracy and decide if an estimate or a measurement is "close enough."	• estimating measurements for various objects or conditions in the environment, such as the Celsius temperature at which it is uncomfortable to go outdoors without a jacket or the metric height of a professional basketball player. The following measurements appeared in a report. Which ones are reasonable and which are not? Explain your choices: • The runner who took first place in the race ran 60 km/hr. • To bake the pizza, we preheated the oven to 200° F. • To bake the pizza, we preheated the oven to 200° C. • A dose of cough syrup is 4 cm. • Two cubic yards of concrete are needed to pour a slab 13 ft. x 12 ft. x 4 in. • developing comparisons of standard units to familiar objects, such as knowing that one gram is approximately the weight of one paper clip. • deciding on appropriate degrees of precision for measurements and their estimates in particular contexts. • comparing the magnitude of the error to the quantity being measured and expressing deviations as a percent of error. • giving examples to illustrate the importance of *percent of error*, such as recognizing that an error of one dollar is a significant overcharge on the purchase of a candy bar, but it is relatively insignificant on the purchase of an automobile. • identifying sources of error or imprecision in measurements. • using informal geometric explorations to develop measurement concepts, such as arranging sectors of a circle to approximate the area, removing and rearranging the vertices of a triangle to find the sum of the angles, or stacking cardboard wafers to approximate volume.

II. Geometry and Measurement

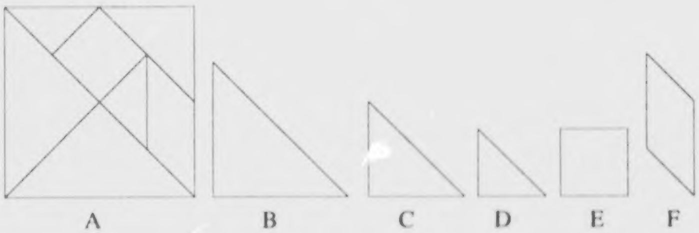
3. Students compare attributes of two objects or of one object with a standard (unit) and analyze situations to determine what measurements should be made and to what level of precision. (Measurement)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
4. Interpret measurements and recognize that two objects may have the same measurement on one attribute (e.g., area), but not necessarily on another (e.g., perimeter).	• using geoboards or dot paper to construct similar figures, and comparing their linear and area measurements. • comparing measurements of sets of objects and drawing conclusions about the dependence or independence of measurements—for example, if two squares have the same area, then they have the same perimeter (and vice versa); however two triangles can have the same area but different perimeters. • expressing relationships in descriptive terms (e.g., if the sides of the similar triangles are in the ratio of 2 to 1, then the areas are in the ratio of 4 to 1).  *"When I doubled (tripled) the length of each side of the triangle, I discovered four (nine) copies of the original figure. The side lengths doubled (tripled), but the area is four (nine) times as great."* • drawing diagrams or making physical models with geoboards or other construction materials to illustrate observed relationships and explain why they are true. • finding the area of a plane shape; cutting the shape into two or more pieces; rearranging the pieces to form a new shape; and comparing the area of the result to the area of the original.

II. Geometry and Measurement

3. Students compare attributes of two objects or of one object with a standard (unit) and analyze situations to determine what measurements should be made and to what level of precision. (Measurement)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
5. Use proportional reasoning and indirect measurements to draw inferences.	• comparing measurements of length, area and volume of similar objects and drawing conclusions about the relationships among these variables.



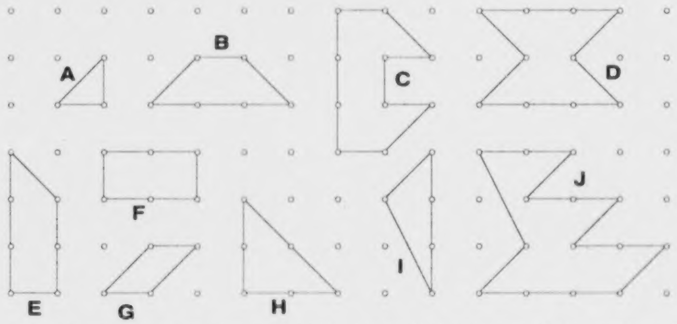
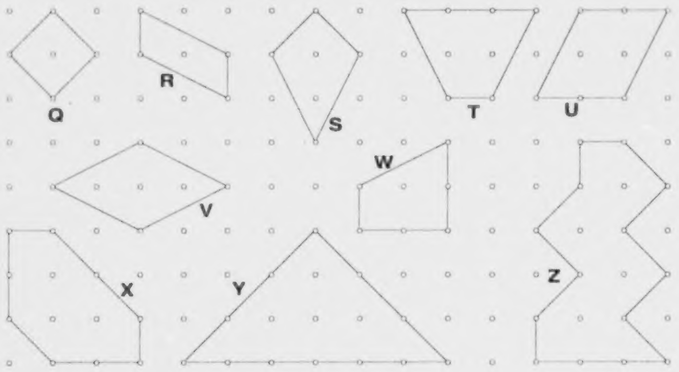
Determine the area of each of the trangram pieces if:

Area of:	A	B	C	D	E	F
			1			
					4	
	4					
		7				
	1					

(continued)

II. Geometry and Measurement

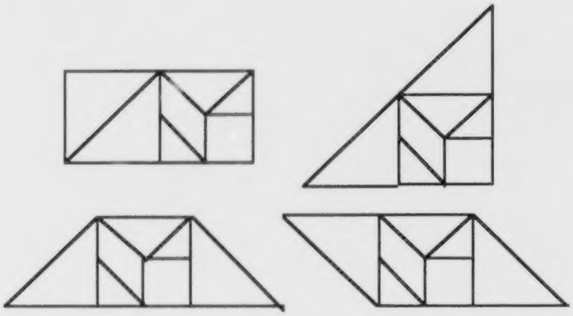
3. Students compare attributes of two objects or of one object with a standard (unit) and analyze situations to determine what measurements should be made and to what level of precision. (Measurement)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
(continued from previous) 5. Use proportional reasoning and indirect measurements to draw inferences.	• deriving strategies for calculating measurements of one quantity from measurements of another (for example, *area of a rectangle* from the measurements of *length* and *width*).  *Copy these shapes on your geoboard. For each shape, record the number of geoboard pegs that lie on the polygon and determine the area of each polygon. Organize your data (number of pegs, area) in a table. Examine your findings and see if you can determine a relationship that tells the area of a geoboard polygon in terms of the number of pegs.* Make some additional polygons of your own and check to see if your rule holds. Also try your rule on the following examples:  Does your rule still hold? If not, can you modify it so that it applies to these cases, too?

(continued)

II. Geometry and Measurement

3. Students compare attributes of two objects or of one object with a standard (unit) and analyze situations to determine what measurements should be made and to what level of precision. (Measurement)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
(continued from previous) 5. Use proportional reasoning and indirect measurements to draw inferences.	• using informal approaches with geoboards, graph paper and other concrete objects to develop and verify basic formulas for area and volume. *Use your tangram pieces to help you find formulas for the area of a rectangle, a triangle, a parallelogram, and a trapezoid. Draw or construct other triangles, parallelograms, and trapezoids and test your formulas on those examples.*  • finding strategies to measure indirectly certain quantities that are very large, very small, or inaccessible, such as measuring the thickness of a book to estimate the thickness of one page, sampling the number of words on several pages to estimate the number of words in a book, or measuring shadows to determine the heights of tall buildings.

II. Geometry and Measurement

3. Students compare attributes of two objects or of one object with a standard (unit) and analyze situations to determine what measurements should be made and to what level of precision. (Measurement)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
6. Apply measurement to describe the real-world and to solve problems.	• experimenting with different-size samples of a given material to observe that, although mass and volume each vary from one sample to another, the ratio of mass to volume (i.e., the *density*) remains constant. • using observed patterns of measurements to predict other measurements (such as predicting the height of a seedling from observed patterns of growth). • comparing the sizes and distances of planets in the solar system, and constructing solar-system models that show both sizes and distances on the same scale. • scale various measurements, such as figuring how high you could jump on the moon or on another planet. • exploring the significance of size in living organisms. • researching how law enforcement officers use measurements of skid marks and other evidence to reconstruct an accident scene.

III. Data Analysis and Statistics

1. Students collect and explore data, organize data into a useful form and develop skill in representing and reading data displayed in different formats.
(Collection, Organization and Presentation of Data)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
1. Collect and explore data through observation, measurement, surveys, sampling techniques and simulations.	• posing questions of interest and importance to them, such as questions about favorite movies or the depletion of natural resources, and gathering data to help answer those questions. • devising and implementing a variety of strategies for collecting data, such as measuring, counting, sampling, surveying, etc. • conducting experiments with random devices (spinners, dice, computer programs, etc.) to generate and explore data.

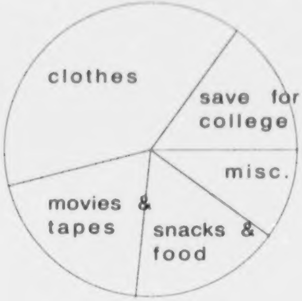
III. Data Analysis and Statistics

1. Students collect and explore data, organize data into a useful form and develop skill in representing and reading data displayed in different formats.
(Collection, Organization and Presentation of Data)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
2. Organize data using tables, charts, graphs, spreadsheets and data bases.	• developing posters, charts, bulletin boards, and other means for presenting data that they have gathered. • selecting examples of different modes of data presentation from newspapers and magazines and describing the information that they contain.

III. Data Analysis and Statistics

1. Students collect and explore data, organize data into a useful form and develop skill in representing and reading data displayed in different formats.
(Collection, Organization and Presentation of Data)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
3. Present data using a variety of appropriate representations, and explain why one representation is preferred over another or how a particular representation may bias the presentation.	• deciding an appropriate way to present data in various situations and explaining their choices (for example, showing why a line graph is preferred to a bar graph or a pie chart in a given setting). "I used a circle graph because I wanted to show parts of the whole." How I Spent My Allowance  • generating examples of situations in which one would choose a particular form of presentation, such as examples of data sets that favor presentation via box plots. • discussing the ways in which data are presented in newspaper and magazine articles, and identifying questions that can be answered from the data. • using a spreadsheet to generate several different presentations from the same data set. • comparing and evaluating several different presentations of the same data. • giving examples of how the presentation of data can be biased, for example by changing the scale of a graph.

III. Data Analysis and Statistics

1. Students collect and explore data, organize data into a useful form and develop skill in representing and reading data displayed in different formats.
(Collection, Organization and Presentation of Data)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
4. Identify what data are needed to answer a particular question or solve a given problem, and design and implement strategies to obtain, organize and present those data.	• identifying everyday situations that rely on data in answering questions, such as predicting election outcomes or making marketing decisions. • identifying interesting problems and asking open-ended questions, identifying the information needed to answer those questions, and devising and implementing data-collection procedures to arrive at appropriate conclusions.

III. Data Analysis and Statistics

2. Students examine data and describe characteristics of the distribution, relate data to the situation from which they arose, and use data to answer questions convincingly and persuasively. (Description and Interpretation)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
1. Critically read data from tables, charts or graphs and explain the source of the data and what the data represent	• examining presentations of data encountered in everyday situations (news reports, advertising campaigns, political speeches, sports statistics, etc.) and raising questions about the data. A major television network reported the following information on its evening news show: Percent of Families with Computers in the Home Whites 27% Blacks 14% Hispanics 13% The anchorman then noted: "Twenty-seven percent of white families have computers at home—as many as black and Hispanic families together. Do you agree with the anchorman's conclusion? Explain. • posing questions about a set of data, and answering the questions posed by other students.

III. Data Analysis and Statistics

2. Students examine data and describe characteristics of the distribution, relate data to the situation from which they arose, and use data to answer questions convincingly and persuasively. (Description and Interpretation)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
2. Describe the shape of a data distribution and identify the center, the spread, correlations, and any outliers.	• discussing the distribution of data using various examples and describing the shapes of those distributions (e.g., rectangular, symmetric, bimodal). • classifying data sets according to selected properties, such as those that are symmetric about their mean. • describing the center of a distribution and explaining how the "center" can be interpreted in different ways, such as the mean vs. the median. • describing special features of various data distributions, such as their spread (range), cluster points, gaps (discontinuities), maximum and minimum values, and outliers. • exploring how certain features of a data distribution can be inferred from visual representations, such as how the length of the box in a box-and-whisker plot conveys information about the spread of the data.

III. Data Analysis and Statistics

2. Students examine data and describe characteristics of the distribution, relate data to the situation from which they arose, and use data to answer questions convincingly and persuasively. (Description and Interpretation)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
3. Draw, explain and justify conclusions based on data.	• preparing presentations that use data to convince the class of a certain conclusion. • writing persuasive letters in which conclusions are presented and supported by data. • staging a class debate in which each side bases its arguments on data it has collected and presented.

II. Geometry and Measurement

2. Students identify locations of objects, location relative to other objects, and the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object. (Position)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
4. Critically question the sources of data; the techniques used to collect, organize and present data; the inferences drawn from the data; and the possible sources of bias in the data or their presentation.	• reading print media and viewing television presentations of data-based conclusions and discussing their validity. • collecting examples of biased presentations and writing rebuttals to expose their biases. • examining how the method of collecting data can influence the outcome, for example by comparing the results of a survey given to a random sample of adults with the results of the same survey given to parents of preschool children. • examining how the size of the sample can influence the outcome of the data gathering. • identifying how data are used in advertising, and raising questions about the sources of those data and the conclusions that the advertisers hope the public will draw.

III. Data Analysis and Statistics

2. Students examine data and describe characteristics of the distribution, relate data to the situation from which they arose, and use data to answer questions convincingly and persuasively. (Description and Interpretation)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
5. Formulate questions and problems, and gather and interpret data to answer those questions.	• raising questions of interest to the students and implementing procedures for gathering data to help answer those questions. • identifying issues affecting students in the school, such as a proposed Student Council project or a change in the dress code, and gathering data as the basis for a report about student preferences.

III. Data Analysis and Statistics

3. Students draw defensible inferences about unknown outcomes; make predictions and identify the degree of confidence they have in their prediction. (Inference and Prediction)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
1. Make and test hypotheses.	• posing interesting questions and generating testable hypotheses to help answer those questions, such as: - What kind of music do students prefer, and which band should we hire for the dance? - If a salad bar is added in the school cafeteria, will the students support it? • generating conjectures about physical situations, such as the amount a spring stretches as weights are suspended from it, and conducting experiments to test those hypotheses.

III. Data Analysis and Statistics

3. Students draw defensible inferences about unknown outcomes; make predictions and identify the degree of confidence they have in their prediction. (Inference and Prediction)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
2. Design experiments to model and solve problems using sampling, simulations, and controlled investigations.	• conducting experiments that model problem situations, such as using dried beans to model a capture-recapture experiment to estimate the fish population in a lake. • exploring phenomena involving several variables, such as conjecturing about the factors (weight of the pendulum bob, length of the pendulum, height from which the pendulum is released) that might determine the period of a pendulum, and designing experiments to systematically control and test variables.

III. Data Analysis and Statistics

3. Students draw defensible inferences about unknown outcomes; make predictions and identify the degree of confidence they have in their prediction. (Inference and Prediction)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
3. Formulate and communicate arguments and conclusions based on data and evaluate their arguments and those of others.	• explaining how they arrived at their conclusions after conducting an investigation. • analyzing the conclusions that other students have put forth and questioning one another about those conclusions. • writing and delivering persuasive arguments designed to convince the listener based on the data presented.

III. Data Analysis and Statistics

3. Students draw defensible inferences about unknown outcomes; make predictions and identify the degree of confidence they have in their prediction. (Inference and Prediction)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
4. Make predictions and decisions based on data, including interpolations and extrapolations.	• looking for patterns in data as a strategy for making predictions. • using data collected in experiments to predict unknown results, and testing those predictions experimentally. • analyzing the sources of the data and the methods used to gather them, and deciding how much confidence they can place in the conclusions drawn from those data. • evaluating the context of a data-gathering situation and determining the extent to which conclusions drawn from the data can be extended to other situations.

III. Data Analysis and Statistics

3. Students draw defensible inferences about unknown outcomes; make predictions and identify the degree of confidence they have in their prediction. (Inference and Prediction)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
5. Employ investigations, mathematical models and simulations to make inferences and predictions to answer questions and solve problems.	• solving problems using data-analysis strategies, such as predicting the "life expectancy" of various boxes of cereal based on a set of factors which the students identify as affecting the consumption rate of cereal. How many people choose this cereal? How much cereal in a serving? How often is the cereal chosen? • conducting simulations to solve problems where actual experiments are impossible, such as to estimate the fish population of a lake or to determine the number of packs of gum they can expect to buy in order to collect an entire set of baseball cards packed with the gum.

IV. Number Sense and Numeration

1. Students experience counting and measuring activities to develop intuitive sense about numbers; develop understanding about properties of numbers; understand the need for and existence of different sets of numbers; and investigate properties of special numbers.
(Concepts and Properties of Numbers)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
1. Develop an understanding of integers and rational numbers, and represent rational numbers in both fraction and decimal form.	• using hands-on manipulative materials to develop number concepts including concepts of place value, prime and composite numbers, fractions and decimals, equivalence, inverses, common factors and common multiples. • using physical models of fractional quantities (e.g., number line, fraction circles, fraction bars) to represent rational quantities such as "fractions close to zero (or 1/2 or 1 ...). • expressing numbers using a variety of equivalent representations, including: - fractions, decimals and percents. - equivalent fractions with common denominators. - physical or geometric representations of fractions or decimals. • expressing mathematical quantities in various ways, such as recognizing that $5 \cdot 94$ can be viewed as $10 \cdot 94 \div 2$ as $\frac{10 \bullet 94}{2}$ as $(5 \cdot 90 + 5 \cdot 4)$, or as $(5 \cdot 100 - 5 \cdot 6)$.

IV. Number Sense and Numeration

1. Students experience counting and measuring activities to develop intuitive sense about numbers; develop understanding about properties of numbers; understand the need for and existence of different sets of numbers; and investigate properties of special numbers.
(Concepts and Properties of Numbers)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
2. Extend their understanding of numeration systems to include decimal numeration, scientific numeration and non-decimal numeration systems.	• exploring very large and very small numbers, and representing such numbers in terms of powers of ten. • constructing concrete representations of various quantities, for example models that illustrate how large is a billion. • exploring selected numeration systems used in other cultures, such as Egyptian or Mayan numerals, as a basis for understanding the power and convenience of a place-value system. • engaging in hands-on activities with multibase blocks, chip trading, or similar materials to explore non-decimal numeration systems. • investigating the use of binary and hexadecimal systems in computing languages. • using concrete models, such as an 8-hour "clock," to develop understanding of modular systems.

IV. Number Sense and Numeration

1. Students experience counting and measuring activities to develop intuitive sense about numbers; develop understanding about properties of numbers; understand the need for and existence of different sets of numbers; and investigate properties of special numbers.
(Concepts and Properties of Numbers)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
3. Develop an understanding of the properties of the integer and rational number systems (e.g., order, density) and of the properties of special numbers including 0, 1, π, and the additive and multiplicative inverses.	• discovering a strategy to illustrate that between any two rational numbers there is always another rational number (density). • arranging sets of numbers, including positive and negative integers and rational numbers, in increasing or decreasing order. • giving examples to illustrate the results of adding or multiplying by zero or one. • measuring circumferences and diameters of round objects to develop a concept of π as the ratio c/d. • demonstrating the behavior of additive and multiplicative inverses, and determining the inverses of specified numbers. • developing strategies for mental computation based on the distributive property, such as recognizing that: $(3.5 \times 9) = (3.5 \times 10) - 3.5 = (4 \times 9) - (.5 \times 9) = 3\cdot 9 + .5\cdot 9$.

IV. Number Sense and Numeration

1. Students experience counting and measuring activities to develop intuitive sense about numbers; develop understanding about properties of numbers; understand the need for and existence of different sets of numbers; and investigate properties of special numbers.
(Concepts and Properties of Numbers)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
4. Apply their understanding of number systems to model and solve mathematical and applied problems.	• constructing numerical expressions to represent problem situations, such as writing an expression for the cost of two 75¢ ice-cream cones and three 60¢ soft drinks, plus tax. • suggesting real applications that correspond to a mathematical expression, such as describing a situation that might be represented by (3 x 17) + 5. • discussing "numerical situations without numbers" such as: Two fractions have a positive sum that is less than one. What can you tell about the fractions? What can you tell about the product of the same two fractions?

IV. Number Sense and Numeration

2. Students recognize that numbers are used in different ways such as counting,measuring, ordering and estimating; understand and produce multiple representations of a number; and translate among equivalent representations. (Representation and Uses of Numbers)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
1. Give geometric representations of fractions, prime and composite numbers, triangular and square numbers, and other number concepts; represent rational numbers and integers on the number line.	• using tiles or markers to represent properties of numbers, such as the geometric arrangements that generate the sequences of triangular and square numbers, or the result that composite numbers can be represented by rectangular arrays in more than one way while prime numbers can only be represented as 1-by-n arrays. • using a number-line model to represent positive and negative integers and rational numbers.

IV. Number Sense and Numeration

2. Students recognize that numbers are used in different ways such as counting,measuring, ordering and estimating; understand and produce multiple representations of a number; and translate among equivalent representations. (Representation and Uses of Numbers)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
2. Recognize equivalent representations of a number, especially fractions, decimals and percents, and translate freely among representations.	• exploring the patterns of repeating decimals and their fractional equivalents. • constructing geometric representations to illustrate how a given fraction, decimal and percent are equivalent representations of the same number. • expressing numbers in scientific notation, and translating numbers from scientific notation to decimal form.

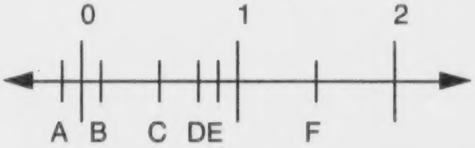
IV. Number Sense and Numeration

2. Students recognize that numbers are used in different ways such as counting,measuring, ordering and estimating; understand and produce multiple representations of a number; and translate among equivalent representations. (Representation and Uses of Numbers)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
3. Distinguish between numbers that are used for counting, numbers that are used for ordering, numbers that are used for measuring, and numbers that are used for naming.	• finding examples in the newspaper of different uses of number, such as counting (e.g., there were 47 people in attendance), ordering (e.g., he placed second in the judging), naming (e.g., she left on her trip aboard a 747), etc. • writing sentences to illustrate different uses of numbers. • illustrating and explaining common uses of numbers, such as free-throw percentages or the use of decimals in batting averages.

IV. Number Sense and Numeration

2. Students recognize that numbers are used in different ways such as counting,measuring, ordering and estimating; understand and produce multiple representations of a number; and translate among equivalent representations. (Representation and Uses of Numbers)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
4. Develop and refine strategies for estimating quantities, including fractional quantities, and evaluate the reasonableness and appropriateness of their estimates.	• approximating the location of fractions and decimals on the number line.  *If the number represented by B is multiplied by 5, what can you say about the result?* *If the number represented by D is multiplied by itself, mark the approximate location of the product.* • estimating the cost of various purchases, including any discounts and sales taxes, before finding the totals with a calculator. • comparing several students' estimates of the same quantity and deciding which is the closest estimate. • verbalizing the strategies they use in arriving at estimates and explaining the effectiveness of their methods. "The paper said that 80,000 persons attended the championship game. Our gym holds 800, so..." • establishing and using benchmarks to estimate numbers and measurements. If I'm 5' 6", He must be about...

IV. Number Sense and Numeration

2. Students recognize that numbers are used in different ways such as counting,measuring, ordering and estimating; understand and produce multiple representations of a number; and translate among equivalent representations. (Representation and Uses of Numbers)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
5. Select appropriate representations for numbers, including integers and rational numbers, in order to simplify and solve problems.	• solving appropriate consumer problems involving discounts, rebates, interest, and taxes. • solving real-life problems involving numbers expressed as fractions, decimals, or percents, or written in scientific notation.

IV. Number Sense and Numeration

3. Students investigate relationships such as equality, inequality, inverses, factors and multiples; and represent and compare very large and very small numbers. (Number Relationships)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
1. Compare and order integers and rational numbers using relations of equality and inequality.	• evaluating expressions without calculating, such as: 318 + 661 + 976 ? 918 + 599 + 268 [<, >, = or ?] 68 x 5 ? 70 + 69 + 68 + 67 + 66 [<, >, = or ?] • ordering fractions by reasoning about their relationship to whole numbers, for example recognizing that 23/24 is greater than 22/23 because, in the first case, the "part not used" (1/24) is less than in the second case (1/23). • locating common irrational numbers on the number line by placing them between two integers (e.g., $\sqrt{5}$ is between 2 and 3) or two rational numbers (e.g., π is between 3.1 and 3.2). • exploring advertised prices and discounts that involve per cents, such as determining how much the customer will save on various purchases or what the discount rate is for an item that is marked down $10. A local craft store ran the following advertisement: **SALE** Your Choice! Take 50% off **OR** Take 25% off Any One Item Your Entire Purchase *How would you decide whether to choose the 50% option or the 25% option? Explain your reasons.*

IV. Number Sense and Numeration

3. Students investigate relationships such as equality, inequality, inverses, factors and multiples; and represent and compare very large and very small numbers. (Number Relationships)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
2. Express numerical comparisons as ratios and rates.	• exploring common applications of rates, such as baseball batting averages, unit pricing or miles-per-hour, and explaining the meaning of such numbers. • using manipulatives to model ratios, such as paper cut-outs to model the ratios between corresponding parts of similar polygons; • exploring simple geometric sequences such as: 1, 3/2, 9/4 , 27/8 ..., and calculating the common ratio of successive terms. • working with maps and scale drawings, and constructing their own scale drawings of selected objects. • graphing linear relationships and explaining how slope represents a rate of change.

IV. Number Sense and Numeration

3. Students investigate relationships such as equality, inequality, inverses, factors and multiples; and represent and compare very large and very small numbers. (Number Relationships)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
3. Distinguish between prime and composite numbers; identify factors, multiples, common factors and multiples, and relatively prime numbers; and apply divisibility tests to numbers.	• developing strategies for determining whether a number is prime, and using a calculator to carry out the investigation. • constructing a systematic method of finding all the factors of a given number with the aid of a calculator. • writing factor trees to illustrate the prime decomposition of various numbers. Two students were factoring the number 48. Natalie wrote 48 = 2 x 24, while Jeremy began with 48 = 6 x 8. If they each develop a factor tree for 48, will they get the same set of prime factors in the end? Can you produce any other factor trees for 48 besides the ones Natalie and Jeremy developed? How many? Did you get the same prime factors? Is what you found out about the prime factors of 48 true for other numbers as well? (Fundamental Theorem of Arithmetic) • exploring common divisibility tests and applying them to selected numbers. Goldbach, a famous mathematician, proposed that any even number greater than 2 can be written as a sum of two prime numbers (for example, 36 = 5 + 31). Pick any 10 even numbers greater than 2 and see if they follow Goldbach's conjecture. Can you find an even number greater than 2 that does not obey this rule • demonstating a procedure for finding multiples of a number. • developing a procedure for finding common factors and common multiples of to or more numbers with the aid of a calculator

IV. Number Sense and Numeration

3. Students investigate relationships such as equality, inequality, inverses, factors and multiples; and represent and compare very large and very small numbers. (Number Relationships)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
4. Explain the meaning of powers and roots of numbers and use calculators to compute powers and square roots.	• exploring the result of repeatedly doubling or halving a quantity, and comparing that to the result of repeatedly adding or subtracting two. • conducting experiments that illustrate exponential growth or decay, such as measuring the heights of successive bounces of a ball. • graphing the results of experiments involving exponential growth or decay. • investigating the relationship between the length of the side and the area of a square, and determining one of the two values when the other is known.

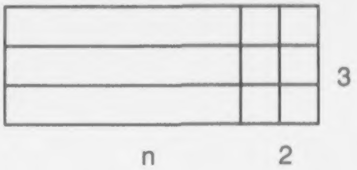
IV. Number Sense and Numeration

3. Students investigate relationships such as equality, inequality, inverses, factors and multiples; and represent and compare very large and very small numbers. (Number Relationships)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
5. Apply their understanding of number relationships in solving problems.	• using published data from a report on contamination in local streams to compare actual conditions to environmental standards for acceptable water quality. • conducting their own environmental studies of air, soil or water quality, radon levels in the home, etc., and writing reports or making presentations on their findings. • monitoring their food and vitamin intake for one day and comparing their consumption to recommended daily allowances.

V. Numerical and Algebraic Operations and Analytical Thinking

1. Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems. (Operations and their Properties)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
1. Use manipulatives and diagrams to model operations and their inverses with integers and rational numbers and relate the models to their symbolic expressions.	• extending familiar models and manipulatives used for operations with whole numbers (number line, multibase blocks, chips, rectangular arrays, etc.) to model operations with fractions, decimals and integers. • writing mathematical expressions to correspond to physica models, for example writing $(n+2)\cdot 3$ to describe.  • drawing pictures to represent mathematical operations, for example a diagram that illustrates (1/3 + 1/2) or (2/3 • 1/4).

V. Numerical and Algebraic Operations and Analytical Thinking

1. Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems. (Operations and their Properties)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
2. Compute with integers, rational numbers and simple algebraic expressions using mental computation, estimation, calculators, and paper-and-pencil; explain what they are doing and know which operations to perform in a given situation.	• developing strategies for estimating computations, for example recognizing that (2.47 x 3.93) is approximately 10 (2.5 · 4). • applying mental estimation to consumer situations, for example reasoning that the bill for 12 assorted candy bars must be less than $6 since each one is less than 50¢. • developing and practicing strategies for mental computation such as combining addends that sum to 10 ($13.30 + $5.95 + $3.70 = $17 + $5.95 = $23 - 5¢) or using related facts ("5+5 =10, so 6+6 would be 12, and then 6+7 would have to be 13"). • comparing the meaning of expressions like (5 + 7) x 3 and 5 + (7 x 3) and explaining the need for an order-of-operations rule for 5 + 7 x 3. • developing facility with a calculator, including the use of memory and special keys, and using a calculator to carry out complex procedures.

V. Numerical and Algebraic Operations and Analytical Thinking

1. Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems. (Operations and their Properties)

<table>
<tr><th>In the middle years, students have the opportunity to...</th><th>By engaging in activities such as...</th></tr>
<tr><td>3. Describe the properties of operations with rationals and integers (e.g., closure; associative, commutative and distributive properties) and give examples of how they use those properties.</td><td>
• using a number line to model the associative and commutative properties for addition with rationals and integers.
• using an area model to demonstrate the associative and commutative properties for multiplication with rationals and integers.
• exploring the distributive property using physical models, diagrams and mathematical expressions.
<table>
<tr><td>3 people (2 books + 3 toys)</td><td>3(2+3) =3(5)</td></tr>
<tr><td>(6 books + 9 toys)</td><td>(6 + 9)</td></tr>
<tr><td>(3 people x (n+2) presents)</td><td>3(n + 2)</td></tr>
</table>
• using manipulatives and diagrams to show that n/n = 1 (n≠0) and using that result to find equivalent fractions by multiplying or dividing by numeral (n/n).
• exploring the relationship between multiplication and division, including multiplication by zero, and offering reasons to explain why division by zero is undefined.
</td></tr>
</table>

V. Numerical and Algebraic Operations and Analytical Thinking

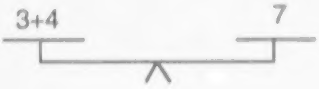
1. Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems. (Operations and their Properties)

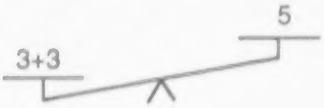
In the middle years, students have the opportunity to...	*By engaging in activities such as...*
4. Efficiently and accurately apply operations with integers, rational numbers, and simple algebraic expressions in solving problems.	• reading and discussing problem situations and explaining what mathematical operations will be needed, and why. • predicting the approximate result of a computation before actually carrying it out. • writing examples of problems that would appropriately be solved with mental computation, paper and pencil, or calculator. • examining the results of several computations and, without calculating, determining which answers are reasonable and which are not.

V. Numerical and Algebraic Operations and Analytical Thinking

2. Students analyze problems to determine an appropriate process for solution and use algebraic notations to model or represent problems. (Algebraic and Analytic Thinking)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
1. Read and write algebraic expressions; develop original examples expressed verbally and algebraically; simplify expressions and translate between verbal and algebraic expressions; and solve linear equations and inequalities.	• writing verbal descriptions of situations that could be represented by given algebraic expressions (e.g., 14x + 37y could represent the total cost of 14 cards and 37 stamps). • writing algebraic expressions and equations that describe selected situations (e.g., How could you express the total cost of tickets for the field trip if adult tickets are $5 and children's tickets are $1.50?). • translating verbal and algebraic expressions that were written by other students. • determining solutions for linear equations and inequalities using informal means including physical models and guess-and-check strategies.





V. Numerical and Algebraic Operations and Analytical Thinking

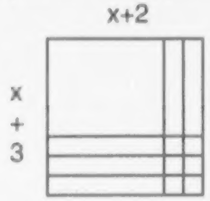
2. Students analyze problems to determine an appropriate process for solution and use algebraic notations to model or represent problems. (Algebraic and Analytic Thinking)

In the middle years, students have the opportunity to...

2. Represent algebraic concepts with geometric models (e.g., algebra tiles), physical models (e.g., balance beam), tables and graphs; and write algebraic expressions to correspond to the multiple representations.

By engaging in activities such as...

- engaging in hands-on explorations with manipulativ es (e.g., algebra tiles, balance beams) and using them to represent algebraic expressions, equations and inequalities.



(x+3)(x+2)=x2+5x=6

- constructing tables of values, including tables generated with computer spreadsheets, to represent algebraic expressions, such as a table of ordered pairs that satisfy the equation m = 3n - 1.

n	1	2	3	4	5
m	2	5	8	11	14

- identifying a pattern in a table of values and using that pattern to construct an algebraic expression to represent the given relationship.

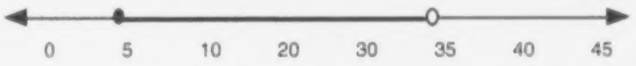
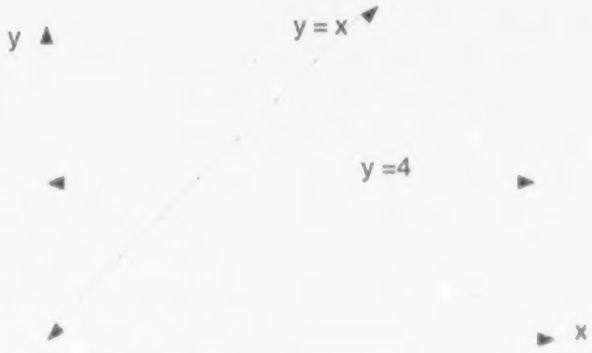
x	1	2	3	4	5
y	2	5	10	17	26

y = ?

- graphing values from a table and explaining how the graph represents the table and the equation derived from that table (e.g., explaining why the graph of entries in a table of circumference vs. diameter of round objects yields a straight line and how the slope of that line relates to the value of pi).
- demonstrating how variables in an algebraic expression are related by showing what happens to one variable when another is changed, for example demonstrating that when the diameter of a circle is tripled, the circumference is also tripled.

V. Numerical and Algebraic Operations and Analytical Thinking

2. Students analyze problems to determine an appropriate process for solution and use algebraic notations to model or represent problems. (Algebraic and Analytic Thinking)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
3. Solve linear equalities and inequalities using algebraic and geometric methods, and use the context of the problem to interpret and explain their solutions.	• using number lines to model solutions to linear equations and inequalities in one variable.  $5 \geq x < 35$ • using geometric representations, including graphs in the Cartesian plane, to model solutions of linear equations in two variables.  • using graphing calculators to locate solutions to linear equations.

V. Numerical and Algebraic Operations and Analytical Thinking

2. Students analyze problems to determine an appropriate process for solution and use algebraic notations to model or represent problems. (Algebraic and Analytic Thinking)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
4. Analyze problems modeled by linear functions, determine strategies for solving the problems, and evaluate the adequacy of the solutions in the context of the problems.	• discussing how individual students understand and represent selected problems and comparing such multiple representations. • devising and explaining their own strategies for solving problems. • discussing whether a solution is acceptable in the context of the problem (for example, 11.69 is an appropriate solution to 456/39 when computing the average cost of 39 items ordered from a catalog—but when computing the number of 39-passenger busses needed to transport 456 students, the answer is 12, while the answer is 11 when determining the number of boxes that can be filled when 456 cookies are packaged in boxes of 39).

V. Numerical and Algebraic Operations and Analytical Thinking

2. Students analyze problems to determine an appropriate process for solution and use algebraic notations to model or represent problems. (Algebraic and Analytic Thinking)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
5. Explore problems that reflect the contemporary uses of mathematics in significant contexts and use the power of technology and algebraic and analytic reasoning to experience the ways mathematics is used in society.	• comparing long-distance calling plans from several different carriers and deciding on the most economical plan to buy. • using a spread sheet to explore the way a savings account grows given different initial investments and rates of interest, and using the information to plan a personal savings strategy.

VI. Probability and Discrete Mathematics

1. Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available; and, make critical judgments about claims that are made in probabilistic situations. (Probability)

In the middle years, students have the opportunity to...	***By engaging in activities such as...***
1. Describe events as likely or unlikely and give qualitative and quantitative descriptions of the degree of likelihood.	• collecting examples of chance and uncertainty drawn from everyday situations and discussing the likelihood of various events. • conducting experiments with simple devices, such as flipping coins or rolling dice, and using the observed results to describe the likelihood of various outcomes. • using technology to generate data for a very large number of events, such as using a computer program to simulate the flipping of a coin 10,000 times—and comparing those results to experiments where a coin was flipped 100 times. • evaluating the decision of a homeowner who failed to buy flood insurance after the 1993 "Flood of the Century" destroyed his property because, he reasoned, "by the time 100 years passes, I'll be long gone."

VI. Probability and Discrete Mathematics

1. Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available; and, make critical judgments about claims that are made in probabilistic situations. (Probability)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
2. Describe probability as a measure of certainty ranging from 0 to 1, and conduct activities that allow them to express probabilities of simple events in mathematical terms.	• examining situations such as rolling two dice, identifying all of the possible outcomes, and describing in their own words what they think is most likely to occur. • drawing a tree diagram to represent the possible outcomes of an experiment and examining the diagram to determine which outcomes are most likely, least likely, equally likely, etc. • writing mathematical expressions to represent the probability of a given event occurring or not occurring, such as the probability of rolling a sum of 4 on two dice. • exploring the meaning of the terms *odds* and *probability*, and comparing "the odds of rolling a sum of 12" with the "probability of rolling a sum of 12" on two dice.

VI. Probability and Discrete Mathematics

1. Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available; and, make critical judgments about claims that are made in probabilistic situations. (Probability)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
3. Conduct experiments and give examples to illustrate the difference between dependent and indepenedent events.	• conducting experiments where items are placed after they are selected compared to expriments where items are not placed after they are selected. • investigating whether the occurence of the first event affects the occurence of the second event.

VI. Probability and Discrete Mathematics

1. Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available; and, make critical judgments about claims that are made in probabilistic situations. (Probability)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
4. Explain the difference between probabilities determined from experiments or chance events (empirical) and probabilities derived mathemematically (theoretical), and explain how the empirical probability changes for a large numbr of trials.	• conducting expirements to generate data that lead to empirical probabilities (such as the probability of rolling "5" on a dice, calculated experimentally by rolling a die 20 times) and comparing those results to the corresponding mathematical or theoretical probability. • combining exprimental data and using computer simulations to produce large numbers of trials, and comparing the probabilities derived from such large samples with those derived from small samples and with the theoretical values.

VI. Probability and Discrete Mathematics

1. Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available; and, make critical judgments about claims that are made in probabilistic situations. (Probability)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
5. Conduct probability expirements and simulations to model and solve problems.	• identifying situations in which probability is used to solve real problems, such as in actuarial applications or product testing. • applying probabilities to model and solve real problems such as using capture-recapture simulations to estimate wildlife populations. • exploring the concept of "fair games" by playing and analyzing simple games and by describing original games.

VI. Probability and Discrete Mathematics

2. Students investigate practical situations such as scheduling, routing,sequencing, networking, organizing and classifying; and analyze ideas like recurrence relations, induction, iteration and algorithm design. (Discrete Mathematics)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
1. Use manipulatives, diagrams, and the fundamental theorem of counting to count permutations and combinations.	• using physical manipulatives to model simple permutation and combination problems. • exploring the fundamental theorem of counting (i.e., if one task can be performed in *m* ways and the second task can be performed in *n* ways, then the number of ways of performing the two tasks is *mn* ways). • observing patterns in the number of permutations of 2, 3, 4, … objects and deriving an expression for the number of possible permutations of any number of objects. • giving examples of situations that illustrate the difference between permutations, where the order of objects is important, and combinations, where order is not important.

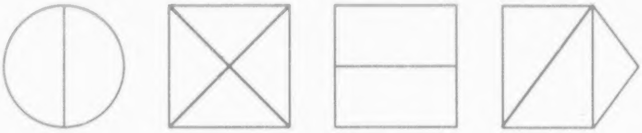
VI. Probability and Discrete Mathematics

2. Students investigate practical situations such as scheduling, routing,sequencing, networking, organizing and classifying; and analyze ideas like recurrence relations, induction, iteration and algorithm design. (Discrete Mathematics)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
2. Use sets and set relationships to explore and solve simple algebraic and geometric problems.	• constructing Venn diagrams to represent mathematical situations (for example, various partitions on the set of integers, or diagrams of the relationships among different sets of quadrilaterals). • describing attributes of given attribute blocks.

VI. Probability and Discrete Mathematics

2. Students investigate practical situations such as scheduling, routing,sequencing, networking, organizing and classifying; and analyze ideas like recurrence relations, induction, iteration and algorithm design. (Discrete Mathematics)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
3. Solve problems involving networks, for example planning delivery routes or counting paths between points.	• tracing paths through a network of vertices and edges.  • using networks to solve problems such as laying out the most efficient route for delivering newspapers.

VI. Probability and Discrete Mathematics

2. Students investigate practical situations such as scheduling, routing,sequencing, networking, organizing and classifying; and analyze ideas like recurrence relations, induction, iteration and algorithm design. (Discrete Mathematics)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
4. Explore recurrence relations and iterations.	• analyzing and extending arithmetic and geometric sequences. • investigating Pascal's triangle and the Fibonacci sequence.

VI. Probability and Discrete Mathematics

2. Students investigate practical situations such as scheduling, routing,sequencing, networking, organizing and classifying; and analyze ideas like recurrence relations, induction, iteration and algorithm design. (Discrete Mathematics)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
5. Continue to use manipulatives and drawings to model the concepts and procedures for the standard arithmetic algorithms, and develop and analyze their own and other students' algorithms to accomplish a task or solve a mathematical problem.	• designing an algorithm to accomplish a task: - design a flow diagram to "assemble" a sandwich. - use a map to write directions to get from point A to point B. • creating and solving logic problems. • investigating the Euclidean algorithm to find the greatest common factor of two numbers.

I. Patterns, Relationships and Functions

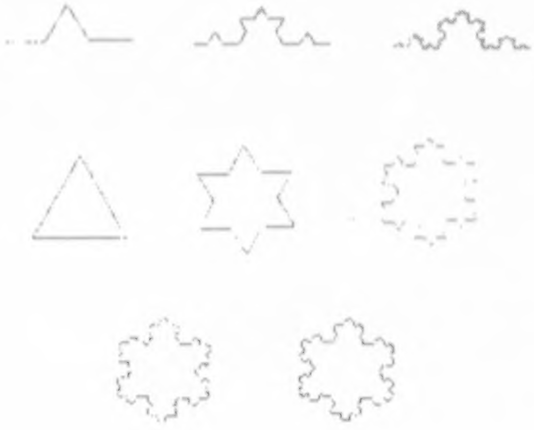
1. Students recognize similarities and generalize patterns; use patterns to create models and make predictions; describe the nature of patterns and relationships,and construct representations of mathematical relationships. (Patterns)

In the middle years, students have the opportunity to...	*By engaging in activities such as...*
6. Use discrete mathematics concepts as described above to model situations and solve problems; and look for whether or not there is a solution (existence problems), determine how many solutions there are (counting problems), and decide upon a best solution (optimization problems).	• using discrete mathematics concepts to model situations and solve problems, such as: - developing schedules for school, buses, television, etc. - creating maps and design delivery routes or rubbish pick-up routes. - using tree diagrams to investigate genetic characteristics. - investigating the concept of "pyramid schemes" or telephone relays. - investigating graphs like the Konigsberg bridge problem. - determining possible combinations of clothing, food, friends, activities, etc. and describe results of adding/subtracting items.

MICHIGAN CURRICULUM FRAMEWORK
MATHEMATICS HIGH SCHOOL ACTIVITIES

I. Patterns, Relationships and Functions

1. Students recognize similarities and generalize patterns; use patterns to create models and make predictions; describe the nature of patterns and relationships, and construct representations of mathematical relationships. (Patterns)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
1. Analyze and generalize mathematical patterns including sequences, series, and recursive patterns.	• analyzing and describing (verbally and symbolically) patterns that arise in various natural settings such as the Fibonacci sequence found in sunflower seedheads and certain leaf arrangements on plants, or symmetry patterns observed in crystals. • exploring and creating geometric patterns similar to those found in fractals like the Koch Snowflake or Sierpinski carpet.  The Koch Snowflake • investigating and generalizing numerical sequences like a geometric, arithmetic, or other sequence (e.g., 1, 3/4, 1/2, 5/16... ,n+1/2n).

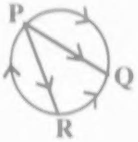
MICHIGAN CURRICULUM FRAMEWORK
MATHEMATICS HIGH SCHOOL ACTIVITIES

I. Patterns, Relationships and Functions

1. Students recognize similarities and generalize patterns; use patterns to create models and make predictions; describe the nature of patterns and relationships, and construct representations of mathematical relationships. (Patterns)

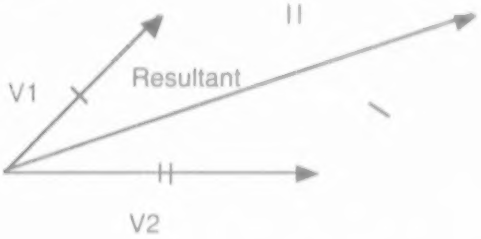
<table>
<tr><th>In the secondary years, students have the opportunity to...</th><th>By engaging in activities such as...</th></tr>
<tr><td>2. Analyze, interpret and translate among representations of patterns including tables, charts, graphs, matrices and vectors.</td><td>

- representing an arithmetic or geometric sequence in recursive form (.5, .25, .125, .0625, ... Æ $a_1 = .5$ and $a_{n+1} = .5a_n$)
- analyzing directed graphs, representing them with adjacency matrices and using the matrix entries to predict the number of paths between pairs of vertices.



$$\Rightarrow \begin{array}{cc} & \begin{array}{ccc} P & Q & R \end{array} \\ \begin{array}{c} P \\ Q \\ R \end{array} & \left[\begin{array}{ccc} 0 & 2 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{array}\right] \end{array}$$

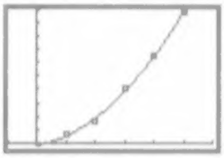
- representing force patterns with vectors and geometrically determining resultant forces.



- constructing an algebraic representation of tabular data (linear, quadratic, exponential, etc.) collected from an experiment or other real-world situation.

<table>
<tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>Y</td><td>3</td><td>8</td><td>20</td><td>33</td><td>49</td></tr>
</table>

$Y = 1.643X^2 + 1.843X - 1$



</td></tr>
</table>

I. Patterns, Relationships and Functions

1. Students recognize similarities and generalize patterns; use patterns to create models and make predictions; describe the nature of patterns and relationships, and construct representations of mathematical relationships. (Patterns)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
3. Study and employ mathematical models of patterns to make inferences, predictions and decisions.	• predicting the smallest number of games an NBA team will need to win to make the playoffs using data from previous years. • deciding which telephone service to buy by analyzing the linear models that represent the base fee and cost per one minute call. Ready Phone Long Distance Service: $8 per month and $0.85 per call Clear Sound Long Distance Service: $11 per month and $0.65 per call *Which is the best deal if you make 35 long distance calls per month?* • using a vertex-edge model to determine a way to connect several cities with fiber optics so that the minimum amount of cable is used.

MICHIGAN CURRICULUM FRAMEWORK
MATHEMATICS HIGH SCHOOL ACTIVITIES

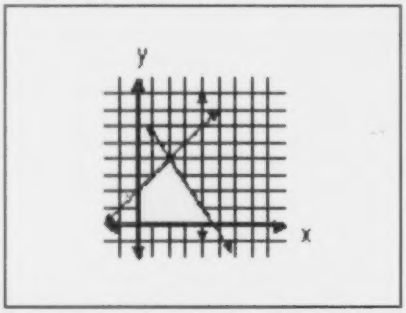
I. Patterns, Relationships and Functions

1. Students recognize similarities and generalize patterns; use patterns to create models and make predictions; describe the nature of patterns and relationships, and construct representations of mathematical relationships. (Patterns)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
4. Explore patterns (graphic, numeric, etc.) characteristic of families of functions; explore structural patterns within systems of objects, operations or relations.	• using the graphics calculator to investigate the effects that parameter changes in the symbolic representations of a function have on the graphs (e.g., changing 'a' in $y = ax^2$, changing 'm' in $y = mx + 4$, changing the b in $y = b^x$, etc.) • exploring the structural patterns within the system of 2 x 2 matrices under multiplication and addition. ✪ investigating the effects of parameter changes on the graphs of the trigonometric functions (e.g., period, amplitude, horizontal and vertical translations).

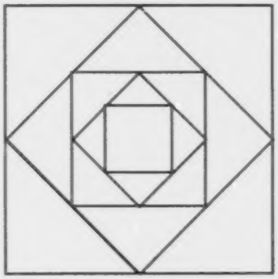
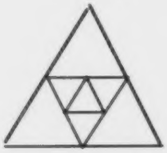
I. Patterns, Relationships and Functions

1. Students recognize similarities and generalize patterns; use patterns to create models and make predictions; describe the nature of patterns and relationships, and construct representations of mathematical relationships. (Patterns)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
5. Use patterns and reasoning to solve problems and explore new content.	• expanding existing mathematical patterns that lead to new mathematical insights as, for example, using the Pythagorean Theorem to derive a formula for the distance between two points or using similar triangles to develop understanding of the trigonometric functions of angles in right triangles • problem solving in which the use of patterns is stressed: - determining the maximum or minimum value in a linear programming situation by testing possible appropriatepairs to generate hypotheses about the location of the solutions.  Explore points that belong to the solution set of this system of inequalities to find the point(s) that would make $f(x,y) = 4x-2y$ have a maximum value. - determining where to build an airport to serve three towns by examining the sum of distances from each town to various sites using a computer-based drawing program. - collecting 5 years of data on the best times for various running events for the school track team and analyzing them to see if patterns in times are evident. ✪ generalizing the sine of an angle in a right triangle to the Law of Sines for any triangle.

I. Patterns, Relationships and Functions

2. Students describe the relationships among variables; predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability and compare patterns of change. (Variability and Change)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
1. Identify and describe the nature of change and begin to use more formal language such as *rate of change, continuity, limits, distribution,* and *deviation.*	• describing change in a variety of mathematical and real-world situations, for instance: - the change that occurs between successive terms of a sequence. - the change that occurs in the areas of the squares in a sequence of squares constructed by making a new square from the previous one by connecting the midpoints of its sides.  - determining the perimeter of a Koch snowflake or the area of the Sierpinski triangle at a given stage. - the variation (deviation in the distribution about the mean) that occurs in univariate and bivariate data collected from various populations of plants, animals or human activities. ⊕ how rate of change in quadratic, exponential or trigonometric. functions is related to the value of the independent variable. • exploring and describing change situations that lead to a limiting position, value or shape (informal limit ideas) as, for instance: - examining the areas of a sequence of equilateral triangles formed by successively joining the midpoints of the sides of each new triangle. 

(continued)

I. Patterns, Relationships and Functions

2. Students describe the relationships among variables; predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability and compare patterns of change. (Variability and Change)

In the secondary years, students have the opportunity to...

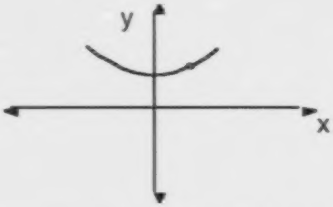
By engaging in activities such as...

(continued from previous page)

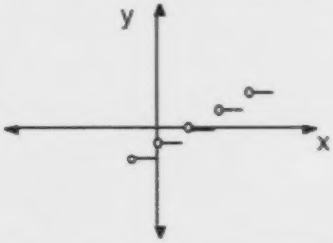
1. Identify and describe the nature of change and begin to use more formal language such as *rate of change, continuity, limits, distribution,* and *deviation.*

- measuring and recording the perimeters of regular polygons with 2^k sides which are inscribed in a circle.
- exploring the asymptotic behavior of graphs like $y = (1/2)^x$
- ⊕ finding partial sums of series to determine if the series converges or diverges.

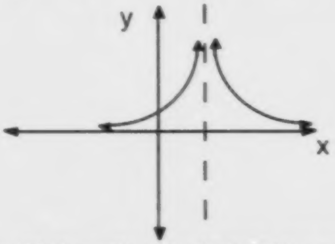
- examining graphs to determine whether they are continuous at all points, and, if not, to describe the nature of the discontinuity, such as:
  - graphs with "holes" in them (point discontinuity).



  - graphs with vertical gaps in them (jump discontinuity).



  - graphs that explode near some value of x (asymptotic discontinuity).



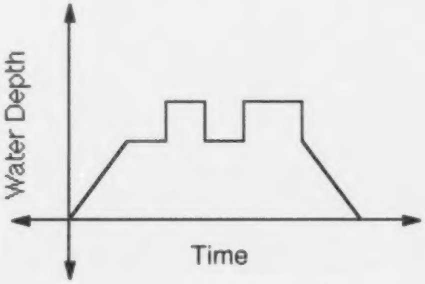
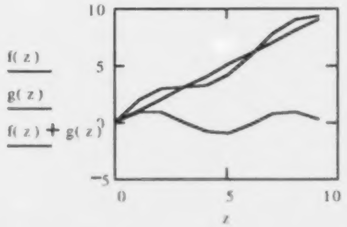
I. Patterns, Relationships and Functions

2. Students describe the relationships among variables; predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability and compare patterns of change. (Variability and Change)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
2. Develop a mathematical concept of function and recognize that functions display characteristic patterns of change (e.g., linear, quadratic, exponential).	• describing the patterns of change characteristic of important functions such as: - a linear function with rule $y = 4x - 2$. - a quadratic function with rule $y = 4x^2 - 2$. - an exponential function with rule $y = 3^x$ - a sine or cosine function with rule $y = \sin x$ or $y = \cos x$. ⊕ a rational function with rule $y = x/(x-2)$. ⊕ all of the six trigonometric functions. • describing the changes evident in scatter plots of bivariate data generated by experiment or found in written material (newspapers, almanacs, texts) such as: - winning times for women's Olympic 100 meter dash. - heights of a bouncing ball as it bounces to a resting position.

I. Patterns, Relationships and Functions

2. Students describe the relationships among variables; predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability and compare patterns of change. (Variability and Change)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
3. Expand their understanding of function to include nonlinear functions, composition of functions, inverses of functions, and piece-wise and recursively defined functions.	• examining patterns in tables, graphs, and equations that illustrate the characteristics of important functions: - nonlinear functions (quadratic, exponential, trigonometric, logarithmic). - composition of functions. - inverses of functions. - piece-wise and recursively defined functions. • describing real-world situations that can be represented by a given graph or sketching graphs of real-world functions, as, for example: - sketching the graph of the day of the year and the time of sunrise. - describing a situation from its graph.  Describe in a paragraph what might have transpired that would make the time versus water depth graph of a hot tub look iike the graph that is pictured. ✪ graphing a product, sum or difference of two functions by using the graphs of the individual functions. The graph illustrates the sum of f(z) and g(z) where f(z)=z and g(z)=sin(z) 

I. Patterns, Relationships and Functions

2. Students describe the relationships among variables; predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability and compare patterns of change. (Variability and Change)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
4. Represent functions using symbolism such as matrices, vectors, and functional representation (f(x)).	• producing a symbolic representation of a function when given appropriate descriptors such as: - using a 2 by 2 matrix for a rotation of magnitude 45 degrees about the origin. - using a vector to represent a translation or displacement with a specified direction and magnitude. - using a functional equation such as $f(t) = -16t^2$ to represent the effect of gravity on a body falling in a vacuum from rest.

I. Patterns, Relationships and Functions

2. Students describe the relationships among variables; predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability and compare patterns of change. (Variability and Change)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
5. Differentiate and analyze classes of functions including linear, power, quadratic, exponential, circular and trigonometric functions and realize that many different situations can be modeled by a particular type of function.	• comparing and contrasting the characteristics of classes of functions. • choosing an appropriate function to model a set of bivariate data such as: - the height of a kicked football as a function of elapsed time. - growth or decay of bacteria, consumption, etc. as a function of elapsed time. - cost of parking a car at the air terminal during a trip. - human biorhythms (sine functions).

I. Patterns, Relationships and Functions

2. Students describe the relationships among variables; predict what will happen to one variable as another variable is changed, analyze natural variation and sources of variability and compare patterns of change. (Variability and Change)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
6. Increase their use of functions and mathematical models to solve problems in context.	• analyzing and modeling situations which are clearly nonlinear: - the height above the ground of a child as she rides a Ferris wheel at the amusement park. - how far, approximately, a batted ball travels if it leaves a bat at 120 feet per second at an angle to the ground of 34 degrees. - the consumption of a natural resource such as coal, timber or radioactive material. - the population growth of an age-specific population like rabbits. • using the functional model of a situation (such as those above) to estimate solutions to questions about the situation.

II. Geometry and Measurement

1. Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes; identify properties and describe relationships among shapes. (Shape and Shape Relationships)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
1. Use shape to identify plane and solid figures, graphs, loci, functions, and data distributions.	• identifying functions, distributions, and geometric patterns visually: - a distribution of the number of boxes of Cracker Jacks one needs to purchase to obtain 6 different prizes which are randomly distributed in the packages (the distribution starts at 6 and tails off to the right). - the symmetries of the frieze pattern of the border of a middle eastern rug (observing symmetrically placed objects). - distinguishing among the graphs of various power functions, given the graphs (e.g. odd powers continue to increase (or decrease) through a flat portion; even powers decrease, flatten and increase or vice versa). • using visual cues to determine possible functional models for scatter-plotted data (do the data seem to be linear? quadratic? exponential?) • identifying three-dimensional shapes like cylinders, prisms, and pyramids that are found in both man-made and naturally occurring objects.

II. Geometry and Measurement

1. Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes; identify properties and describe relationships among shapes.
(Shape and Shape Relationships)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
2. Determine necessary and sufficient conditions for the existence of a particular shape and apply those conditions to analyze shapes	• using the properties of a plane shape to design an algorithm for constructing the shape using both traditional and computer-assisted methods, such as: - designing a procedure for constructing a parallelogram. - finding a strategy to bisect a line segment. • using the characteristics of plane shapes to identify specific shapes or properties such as: - the quadrilaterals whose diagonals are perpendicular. - the quadrilaterals that can be inscribed in a circle. - additional characteristics of shapes. *Draw an isosceles triangle using an automatic drawing program and bisect the vertex angle. Record the measurements of all angles and segment lengths within the drawing. Use the program to change to several other different sizes of isosceles triangles and record the same measurements as you did for the first triangle. Can you make any conjectures about other properties that appear to hold for isosceles triangles?*

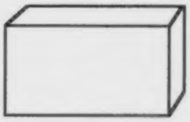
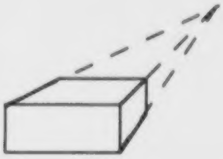
II. Geometry and Measurement

1. Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes; identify properties and describe relationships among shapes. (Shape and Shape Relationships)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
3. Use transformational, coordinate or synthetic methods to verify (prove) the generalizations they have made about properties of classes of shapes.	• deducing properties of shapes or relationships among shapes using a variety of representations such as: - deducing familiar triangle congruence and similarity theorems using transformation techniques. - deducing characteristics of quadrilaterals using coordinate techniques (distance, slope, midpoints, etc.). - deducing characteristics of parallelograms synthetically using congruent triangles and a standard definition of a parallelogram.

II. Geometry and Measurement

1. Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes; identify properties and describe relationships among shapes.
(Shape and Shape Relationships)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
4. Draw and construct shapes in two- and three- dimensions and analyze and justify the steps of their constructions.	• constructing two and three dimensional shapes using a variety of tools: - compass and straightedge constructions of plane shapes. - scissors, paper and glue constructions of models of three dimensional shapes. - scripts for computer drawing utilities that reproduce constructions over a variety of shapes in a given class. • proving algorithms for constructions of plane shapes. • sketching two dimensional representations of solid shapes: - solids sketched with the hidden edges and faces shown and parallelism maintained.  - using perspective to sketch solids as viewed from one-point perspective.  ⊕ common solids and solids of revolution sketched within a three dimensional rectangular coordinate system with key coordinates affixed.

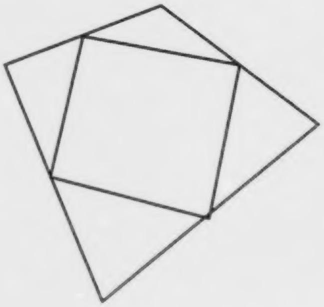
II. Geometry and Measurement

1. Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes; identify properties and describe relationships among shapes.
(Shape and Shape Relationships)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
5. Study transformations of shapes using isometries, size transformations and coordinate mappings.	• determining the invariances of a class of transformations in the plane such as: - rigid motions (the isometries) both experimentally and formally. - size transformations (dilations) in a coordinate environment using the origin as center and (x,y) mapping onto (kx,ky) for some magnitude k. ✪ investigating the effects of coordinate transformations like (x,y) maps to (x+y, x) on distance, angle, shape and collinearity.

II. Geometry and Measurement

1. Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes; identify properties and describe relationships among shapes.
(Shape and Shape Relationships)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
6. Compare and analyze shapes and formally establish the relationships among them, including congruence, similarity, parallelism, perpendicularity and incidence.	• examining two or more shapes for possible relationships among them, such as: - two triangles may be congruent or similar. - two lines may be perpendicular, parallel, or obliquely intersecting. - three lines containing the vertices of a triangle may have a point in common (incidence of medians, angle bisectors, and altitudes) • using a variety of techniques to justify conjectures about relations among two or more shapes. • observing and proving the nature of a quadrilateral constructed by joining the midpoints of the sides of any other quadrilateral. 

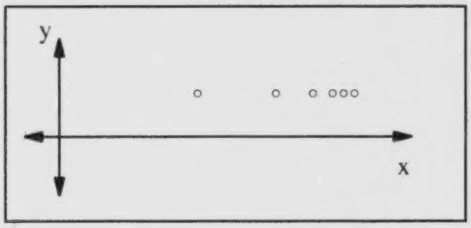
II. Geometry and Measurement

1. Students develop spatial sense, use shape as an analytic and descriptive tool, identify characteristics and define shapes; identify properties and describe relationships among shapes.
(Shape and Shape Relationships)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
7. Use shape, shape properties, and shape relationships to describe objects and to solve problems.	• using shape concepts to help make sense of observations in business, science, sports, and the environment, such as: - the characteristics of frieze patterns found in architecture, art and crafts. - the characteristics of data collected from a science experiment (e.g., spring stretching, pendulum swinging, etc.). - the characteristics of honeycomb cells (optimization of volume when compared to other prism shapes). - the predicted future Olympic winning times and scores based on models derived from the historical data. - using the normal curve to explain quality control and rare or common occurrences.

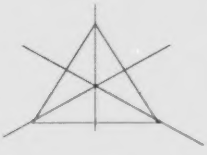
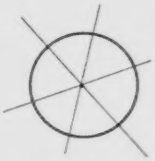
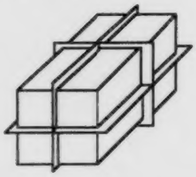
II. Geometry and Measurement

2. Students identify locations of objects, location relative to other objects, and the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object.(Position)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
1. Locate and describe objects in terms of their position, including polar coordinates, three-dimensional Cartesian coordinates, vectors, and limits.	• using position to locate and describe mathematical objects such as: - modeling a sequence of numbers graphically to illustrate the idea of a limiting position.  - graphing points on two and three dimensional coordinate systems. - locating a moving point by finding the resultant of two or more linear motions represented by vectors. • using parametric equations to represent linear and non-linear motions. ✪ graphing points and functions in the polar plane.

II. Geometry and Measurement

2. Students identify locations of objects, location relative to other objects, and the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object.(Position)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
2. Locate and describe objects in terms of their orientation and relative position, including displacement (vectors), phase shift, maxima, minima and inflection points; give precise mathematical descriptions of symmetries.	• using a computer or graphics calculator and the concepts of orientation and relative position to locate and describe: - important points (e.g., intercepts, relative maxima and minima) on a function graph. - key points on statistical graphs (e.g., median and quartiles on box plot) - images and preimages of transformed figures. ⊕ points of inflection and phase shifts for graphs of functions. • exploring triangles to find orthocenters, incenters, centroids etc., using a compass, a reflective tool or computer constructions to assist conjecture development. • exploring and describing symmetries of both two- and three-dimensional figures. equilateral triangle - 3 lines of symmetry  circle - infinite lines of symmetry  rectangular solid - 3 planes of symmetry 

II. Geometry and Measurement

2. Students identify locations of objects, location relative to other objects, and the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object.(Position)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
3. Give precise mathematical descriptions of transformations and describe the effects of transformations on size, shape, position and orientation.	• exploring and describing the effects of transformations on the size, shape, position, collinearity, betweenness and orientation of objects using a variety of methods and tools: - computer geometry drawing utilities. - coordinate graphing. - reflecting tools, rulers, compasses and protractors. • describing transformations using geometric or algebraic methods, such as: - size transformations using coordinates. - rotations using matrices. - translations using vectors. - isometries and size transformations synthetically.

II. Geometry and Measurement

2. Students identify locations of objects, location relative to other objects, and the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object.(Position)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
4. Describe the locus of a point by a rule or mathematical expression; trace the locus of a moving point.	• describing the position (locus) of a point satisfying two or more conditions in a variety of ways: - synthetically as the intersection or union of geometric point sets (equidistant from two lines or equidistant from two points). - algebraically, for instance, giving the height of a particle moving vertically with or without forces other than gravity. - parametrically giving, for instance, the position and path of a projectile launched at any angle to the horizontal (a batted softball).

II. Geometry and Measurement

2. Students identify locations of objects, location relative to other objects, and the effects of transformations (e.g., sliding, flipping, turning, enlarging, reducing) on an object.(Position)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
5. Use concepts of position, direction and orientation to describe the physical world and to solve problems. -	• investigating, analyzing and describing situations such as: - projectile motion. - courses for air and water crafts. - the path of an object subject to several competing forces. - position on a planar or spherical map. - the path of an object subject to several competing forces.

II. Geometry and Measurement

3. Students compare attributes of two objects or of one object with a standard (unit) and analyze situations to determine what measurements should be made and to what level of precision. (Measurement)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
1. Select and use appropriate tools; make accurate measurements using both metric and common units; and measure angles in degrees and radians.	• continuing to use appropriate measuring tools skillfully to make measurements such as: - making lengths and angles in terms of a given length or angle using a compass. - measuring angles in degrees and radians to draw describe the direction of a vector, determine the angle of a rotation, or the supplement of an angle. - using various probes with a computer to measure motion, force, sound, pressure, temperature, etc. ✪ sketching an angle of one radian and describing how its accuracy can be checked by using a circle with radius *r*.

II. Geometry and Measurement

3. Students compare attributes of two objects or of one object with a standard (unit) and analyze situations to determine what measurements should be made and to what level of precision. (Measurement)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
2. Continue to make and apply measurements of length, mass (weight), time, temperature, area, volume, angle; and classify objects according to their dimensions.	• choosing and applying a unit of measure appropriate to the object or attribute being measured. • categorizing geometric objects with respect to their dimension such as: - one dimensional segments. - two dimensional plane shapes. - three dimensional space shapes. ⊕ geometric fractals that have dimensions that are between 0 and 3. (Sierpinski triangle and carpet, Koch snowflake, Norway's coastline, etc.) • using measurements in applied situations (industrial arts, physics, biology, chemistry, foods, etc.)

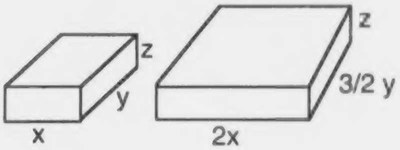
II. Geometry and Measurement

3. Students compare attributes of two objects or of one object with a standard (unit) and analyze situations to determine what measurements should be made and to what level of precision. (Measurement)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
3. Estimate measures with a specified degree of accuracy and evaluate measurements for accuracy, precision and tolerance.	• continuing to estimate measures of 1, 2, and 3 dimensional objects. • recognizing that all measurements from observations or experiments involve numerical results that contain some degree of error and estimating the errors incurred in making those measurements: - per cent error: the difference between a measured or observed value and the accepted value as a percent of the accepted value. $\text{per cent error} = \frac{\text{accepted - measured}}{\text{accepted value}} \bullet 100\%$ - per cent difference: difference in two measures of the same attribute as a per cent of the average measure (a measure of relative error). $\text{per cent difference} = \frac{\text{difference in measures}}{\text{average measure}} \bullet 100\%$ • specifying the level of accuracy or precision needed in a given measurement situation: - precision needed in grinding the cylinder of an engine. - precision needed in cutting a board for a book shelf. • recognizing and estimating the effect of small measurement errors on values computed from the measures, such as an area or volume when edges of an object are measured.

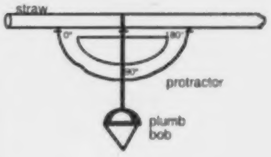
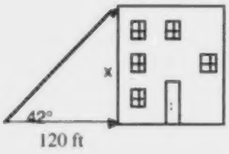
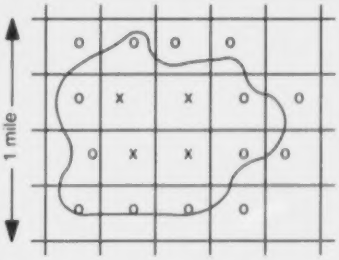
II. Geometry and Measurement

3. Students compare attributes of two objects or of one object with a standard (unit) and analyze situations to determine what measurements should be made and to what level of precision. (Measurement)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
4. Interpret measurements and explain how changes in one measure may affect other measures.	• investigating and generalizing patterns found in measures of area and volume when the length of one or more edges of a two or three dimensional object is changed, such as: - the effect on the area of halving or tripling all the sides of a plane shape. - the effect on the surface area and volume when the lengths of any four parallel edges of a box are changed.  S.A. = $2xz + 2yz + 2xy$ → S.A. = $4xz + 3yz + 6xy$ Surface Area increases by $2xy + yz + 4xy$ $V = xyz$ → $V = 3xyz$ Volume is tripled - comparing the effects of additive changes to the effects of multiplicative changes, such as adding 3 to each side of a rectangle compared to multiplying each side by 3. • investigating the relationships among the corresponding lengths, areas and volumes of similar geometric figures or physical objects. • investigating the relationships between areas and perimeters of plane shapes and between volúmes and surface areas of solids, such as: - the areas of different rectangles with a given perimeter. - the volumes of prisms with a given surface area. • exploring rectangles of a given perimeter to see if a maximum area can be found.

II. Geometry and Measurement

3. Students compare attributes of two objects or of one object with a standard (unit) and analyze situations to determine what measurements should be made and to what level of precision. (Measurement)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
5. Use proportional reasoning and indirect measurements, including applications of trigonometric ratios, to measure inaccessible distances and to determine derived measures such as density.	• using proportional reasoning and similarity to explain real phenomena such as: - the impossibility of the existence of giants similar to ourselves. - insects that walk on water. - thoroughbred horses' tendency to break legs while running. • using a homemade sighting device to assist in indirectly measuring physical objects such as: - the distance from an object of known height. - the height of school building or flag pole.   $\tan 42° = x / 120$ $120 \tan 42° = x$ $108 \text{ ft} \approx x$ • using grids with varying unit dimensions to measure the area of objects with irregular shapes to any degree of precision.  x = 4 full squares o = 14 partial squares each suare is $1/4 \cdot 1/4 = 1/16$ sq. mi Area $= (4 + 1/2 \cdot 14) \cdot 1/16$ $= 11/16$ square miles • using the Law of Sines and the Law of Cosines to find measures of inaccessible distances and angles.

II. Geometry and Measurement

3. Students compare attributes of two objects or of one object with a standard (unit) and analyze situations to determine what measurements should be made and to what level of precision. (Measurement)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
6. Apply measurement to describe the real-world and to solve problems.	• using measurement to describe real-world situations such as: - investigating the possible composition of an object by determining its density (mass /volume). - measuring the leaf length versus the stem length for maple leaves from the same tree, entering the data into a graphics calculator or computer, creating a scatterplot, and determining if there is a relationship between the two measures. • using a computer with probes to gather and analyze measurements of physical phenomena: - relating a ball's rebound height to the number of times it has bounced. - using a motion detector to determine the terminal velocity of a falling object and comparing the findings with the mathematical model for terminal velocity. - using force sensors to measure the effect of addition of force vectors. • using a distribution of measures of an object to assign the most appropriate measure of the attribute being measured. • investigating the special measurement tools and techniques used by various professions, such as forestry, surveying, medical technology, and engineering.

III. Data Analysis and Statistics

1. Students collect and explore data, organize data into a useful form and develop skill in representing and reading data displayed in different formats.
(Collection, Organization and Presentation of Data)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
1. Collect and explore data through observation, measurement, surveys, sampling techniques and simulations.	• designing and administering data collection strategies to answer questions important to the learners as, for example: - those related to student views of the school environment (lunchroom use, sports, alcohol use, academic offerings) using surveys. - those related to physical phenomena (drop height vs. rebound height of ball, period vs. pendulum length, time vs. temperature of moth balls heated to melting and allowed to cool) using experiments. - those situations that can be probabilistically modeled (e.g.,number of boxes of cereal needed to purchase to get complete collection of toys) using probability simulations.

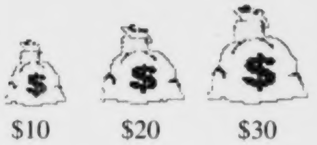
III. Data Analysis and Statistics

1. Students collect and explore data, organize data into a useful form and develop skill in representing and reading data displayed in different formats.
(Collection, Organization and Presentation of Data)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
2. Organize data using tables, charts, graphs, spreadsheets and data bases.	• organizing data to highlight their essential characteristics in an appropriate form such as in: - tables. - box plots. - scatter plots. - spreadsheets. - stem-and-leaf plots. - histograms.

III. Data Analysis and Statistics

1. Students collect and explore data, organize data into a useful form and develop skill in representing and reading data displayed in different formats.
(Collection, Organization and Presentation of Data)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
3. Present data using the most appropriate representation and give a rationale for their choice; show how certain representations may skew the data or bias the presentation.	• choosing, presenting and defending the form used to represent a given set of data. • critiquing data presented in newspapers, almanacs and other sources considering issues such as: - how the data were collected. - who collected the data. - the possibility and nature of any bias. - the appropriateness of the presentation form.  Carefully look over the data presentation at the left and write a paragraph telling whether you think it is a good choice of representation or not and explain your reasoning.

III. Data Analysis and Statistics

1. Students collect and explore data, organize data into a useful form and develop skill in representing and reading data displayed in different formats.
(Collection, Organization and Presentation of Data)

In the secondary years, students have the opportunity to...

4. Identify what data are needed to answer a particular question or solve a given problem, and design and implement strategies to obtain, organize and present those data.

By engaging in activities such as...

- generating questions about social, ethical, political, religious, scientific, environmental, etc. issues for which data collection and analysis may provide information regarding possible solutions, such as:
  - what is the effect on China's population of its policy of allowing families to have only one child?;
  - what are the views of your schoolmates regarding the opportunities for club activities at your school?
- determining and implementing strategies to collect data associated with interesting questions:
  - using coin tosses to simulate child birth in China.
  - constructing a bias-free survey to determine student views
- organizing and presenting data in an appropriate form.

Below are the bowling scores from the final tournament for the top three teens in the league:

Jennifer	*Michelle*	*Krista*
200	*173*	*202*
197	*172*	*156*
187	*198*	*192*
185	*184*	*187*
189	*181*	*182*

Organize the data and choose an appropriate representation that will best show a comparison among the girls' scores.

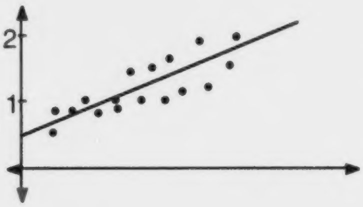
III. Data Analysis and Statistics

2. Students examine data and describe characteristics of the distribution; and, they relate data to the situation from which they arose and use data to answer questions convincingly and persuasively. (Description and Interpretation)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
1. Critically read data from tables, charts or graphs and explain the source of the data and what the data represent.	• reading and interpreting data found in a variety of sources such as newspapers, almanacs, government reports and presented in all of the common formats. • describing the sources of data and how both the data and the sources provide information important to intelligent use of the data.

III. Data Analysis and Statistics

2. Students examine data and describe characteristics of the distribution; and, they relate data to the situation from which they arose and use data to answer questions convincingly and persuasively. (Description and Interpretation)

In the early years, children have the opportunity to...	***By engaging in activities such as...***
2. Describe the shape of a data distribution and determine measures of central tendency, variability and correlation.	• describing the shape of a distribution in terms of symmetry and skewness and in relation to a normal distribution. • using calculator and/or computer technology to find measures of central tendency and variability of univariate data. • using visual displays to estimate the association between two variables.  Variables appear to have a correlation coefficient greater than .75 and look like they could be approximated by the line $y = .5x + .5$ • employing calculator or computer technology to calculate the correlation coefficient for data that appear linear.

III. Data Analysis and Statistics

2. Students examine data and describe characteristics of the distribution; and, they relate data to the situation from which they arose and use data to answer questions convincingly and persuasively. (Description and Interpretation)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
3. Use the data and their characteristics to draw and support conclusions.	• determining and testing the probable best regression model for bivariate data for situations such as the following: - height of drop vs. rebound height of a ball (linear). - period vs. string length of a pendulum (non-linear). - number of dispensed cup of coffee vs. time to fill from an urn (quadratic). - length of an image on a projection screen vs. length of preimage (linear). - newspaper- and almanac-published data sets or charts. • using measures of central tendency, variability, and appropriate regression models to describe a sample and its typical elements.

III. Data Analysis and Statistics

2. Students examine data and describe characteristics of the distribution; and, they relate data to the situation from which they arose and use data to answer questions convincingly and persuasively. (Description and Interpretation)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
4. Critically question the sources of data; the techniques used to collect, organize and present data; the inferences drawn from the data; and the sources of bias and measures taken to eliminate such bias.	• examining data sources and inferences drawn from them for possible bias and for the appropriate use of common statistics. • representing data in other familiar forms to determine if inferences made on the basis of one representation remain reasonable, e.g., visually examining a scatter plot before using a regression equation to calculate predicted values. • identifying sources of bias in data sets in situations such as: - data collected by student surveys. - data presented in public business or government documents. - data generated by measuring physical phenomena (e.g., height of bouncing ball, duration of pendulum swing). • describing and designing strategies for reducing bias such as: - making response items neutral in surveys. - requiring clear statements of collection procedures and populations used in business and government document data. - improving the measurement techniques used to collect physical data.

III. Data Analysis and Statistics

2. Students examine data and describe characteristics of the distribution; and, they relate data to the situation from which they arose and use data to answer questions convincingly and persuasively. (Description and Interpretation)

In the early years, children have the opportunity to...	*By engaging in activities such as...*
5. Formulate questions and problems, and interpret data to answer those questions.	• generating questions and collecting and interpreting data about social, ethical, political, religious, scientific, environmental, etc. issues, such as: - the number of meals at a fast-food chain one would need to purchase to obtain all 5 possible gifts if the gifts are randomly provided with each meal purchase. - the views of your schoolmates regarding the availability of varsity sports for boys and for girls. - the number of light bulbs you would have to buy to get aninferior one if inferior ones make up about 0.1 percent of those manufactured.

III. Data Analysis and Statistics

3. Students draw defensible inferences about unknown outcomes; make predictions and identify the degree of confidence they have in their prediction. (Inference and Prediction)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
1. Make and test hypotheses.	• making and testing hypotheses related to social, ethical, political, scientific, environmental, mathematical, school and peer concerns: - what criteria do classmates use to decide the student council candidates for whom they vote? - what would a person do if (s)he found a wallet containing $50 and clear identification of the owner? - how is the population of your community changing? - what effect does changing the coefficient of the squared term of a quadratic have on its graphical representation? - how does changing the slope of a ramp affect the time it takes a matchbook car to get to the bottom?

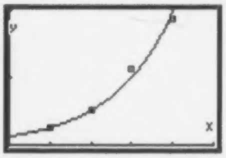
III. Data Analysis and Statistics

3. Students draw defensible inferences about unknown outcomes; make predictions and identify the degree of confidence they have in their prediction. (Inference and Prediction)

<table>
<tr><th><i>In the secondary years, students have the opportunity to...</i></th><th><i>By engaging in activities such as...</i></th></tr>
<tr><td>2. Design investigations to model and solve problems; also employ confidence intervals and curve fitting in analyzing the data.</td><td>

- designing experiments and investigations to gather data that can be used to inform views about particular questions such as those listed above.
- when the data permit it, using curve-fitted functions to assist in the analysis of the data.

<table>
<tr><th>X</th><th>1</th><th>2</th><th>3</th><th>4</th></tr>
<tr><td>Y</td><td>2</td><td>4</td><td>9</td><td>15</td></tr>
</table>



$Y = 1.033(1.985)^X$

- using confidence intervals to specify the degree of confidence in a proposed solution to a problem.

</td></tr>
</table>

III. Data Analysis and Statistics

3. Students draw defensible inferences about unknown outcomes; make predictions and identify the degree of confidence they have in their prediction. (Inference and Prediction)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
3. Formulate and communicate arguments and conclusions based on data and evaluate their arguments and those of others.	• formulating, communicating and evaluating conclusions derived from information about issues generated from experiments, investigations, simulations or from information found in newspapers, almanacs, magazines, etc. • informally determining confidence levels for inferences made from survey data based on possible biasing factors, such as: - when the survey was conducted. - who conducted the survey. - who sponsored the survey. - the size of the sample. - the methods of collection.

III. Data Analysis and Statistics

3. Students draw defensible inferences about unknown outcomes; make predictions and identify the degree of confidence they have in their prediction. (Inference and Prediction)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
4. Make predictions and decisions based on data, including interpolations and extrapolations.	• using appropriate regression models of bivariate data to make predictions of future events (extrapolations) or to estimate values intermediate to those given (interpolation) such as: - predicting the population of Michigan based on population data for 10 year intervals. - predicting the winning time for the next women's Olympic 200 meter run based on previous winning times. - predicting the number of deer in the Oceana County herd on the basis of a "tag and recapture" technique of sampling. - predicting the time elapsed since an animal died on the basis of the temperature of the body.

III. Data Analysis and Statistics

3. Students draw defensible inferences about unknown outcomes; make predictions and identify the degree of confidence they have in their prediction. (Inference and Prediction)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
5. Employ investigations, mathematical models and simulations to make inferences and predictions to answer questions and solve problems.	• predicting the winner of a school election on the basis of the votes of a random sample of student voters. • predicting a typical number of draws necessary to get at least one of each of 7 prizes randomly placed in cereal boxes or fast food children's meals. • predicting the average wait time needed to get through the supermarket check-out line. • predicting the amount of time a natural resource (coal, oil, timber, etc.) would last given a mathematical model of its consumption. • predicting animal population using a "tag and recapture" technique.

IV. Number Sense and Numeration

1. Students experience counting and measuring activities to develop intuitive sense about numbers; develop understanding about properties of numbers; understand the need for and existence of different sets of numbers; and investigate properties of special numbers.
(Concepts and Properties of Numbers)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
1. Develop an understanding of irrational, real and complex numbers.	• discussing the nature of the solutions that can arise from quadratic equations such as: - irrational solutions to equations like $x^2 = 7$. - complex solutions to equations like $x^2 = -7$. • identifying the subset relationships within the complex numbers. • explaining and illustrating the similarities and differences among the types of numbers in the real number system (irrational, rational, etc.) ✪ representing complex numbers in $(a + bi)$ or $r(\cos\theta + i\sin\theta)$ form, and as ordered pairs (a,b) or as (r,θ) in the complex plane.

IV. Number Sense and Numeration

1. Students experience counting and measuring activities to develop intuitive sense about numbers; develop understanding about properties of numbers; understand the need for and existence of different sets of numbers; and investigate properties of special numbers.
(Concepts and Properties of Numbers)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
2. Use the (a+bi) and polar forms of complex numbers.	• identifying the real and imaginary parts of complex numbers. ✪ operating on complex numbers in (a+ bi) and polar form and translating between the two forms.

IV. Number Sense and Numeration

1. Students experience counting and measuring activities to develop intuitive sense about numbers; develop understanding about properties of numbers; understand the need for and existence of different sets of numbers; and investigate properties of special numbers.
(Concepts and Properties of Numbers)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
3. Develop an understanding of the properties of the real and complex number systems and of the properties of special numbers including π, i, e, and conjugates.	• investigating the values and properties of special numbers such as: - the number π using a Monte Carlo method on a computer or graphics calculator. - the Euler number ***e***, found in many business and scientific applications, by evaluating the expression $(1 + 1/n)^n$ for successively larger values of n with calculator or computer assistance. - the number ***i*** when defined as the $\sqrt{-1}$. - the use of π in representing angles in radian measure. ⊕ the imaginary number ***i*** by exploring patterns in successive powers of ***i***. ⊕ conjugates of complex numbers symbolically and graphically. ✪ exploring group and field properties and proving elementary theorems within those structures. ✪ investigating similarities and differences between simple mathematical systems, such as square matrices and the set of integers under the operations of addition or multiplication.

IV. Number Sense and Numeration

1. Students experience counting and measuring activities to develop intuitive sense about numbers; develop understanding about properties of numbers; understand the need for and existence of different sets of numbers; and investigate properties of special numbers.
(Concepts and Properties of Numbers)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
4. Apply their understanding of number systems to model and solve mathematical and applied problems.	• investigating the effects of using rational approximations for real numbers: - how does a calculator, which must use rational approximations, avoid large computational error when computing with the approximations? - how does the engineer determine the dimensions of a 3 cm^2 solar collector that will ensure the desired surface area yet be as small as possible?

IV. Number Sense and Numeration

2. Students recognize that numbers are used in different ways like counting, measuring, ordering and estimating; understand and produce multiple representations of a number; and translate among equivalent representations. (Representation and Uses of Numbers)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
1. Give decimal representations of rational and irrational numbers and coordinate and vector representations of complex numbers.	• writing decimal representations for rational and irrational numbers to a desired degree of accuracy. • representing complex numbers geometrically with vectors in a rectangular coordinate system. ✪ representing complex numbers in polar coordinate form.

IV. Number Sense and Numeration

2. Students recognize that numbers are used in different ways like counting, measuring, ordering and estimating; understand and produce multiple representations of a number; and translate among equivalent representations. (Representation and Uses of Numbers)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
2. Develop an understanding of more complex representations of numbers, including exponential and logarithmic expressions, and select an appropriate representation to facilitate problem solving.	• constructing geometric and numerical representations of numbers written exponentially. • observing patterns in data generated by a calculator or computer to develop rules for operating with exponents and logarithms.

$\log 3 \approx .477$	$\log 19 \approx 1.279$	$\log 12 \approx 1.079$
$\log 4 \approx .602$	$\log 10 = 1$	$\log 12 \approx 1.079$
$\log 12 \approx 1.079$	$\log 190 \approx 2.279$	$\log 144 \approx 2.158$

$\rightarrow \log(a \bullet b) = \log a + \log b$

IV. Number Sense and Numeration

2. Students recognize that numbers are used in different ways like counting, measuring, ordering and estimating; understand and produce multiple representations of a number; and translate among equivalent representations. (Representation and Uses of Numbers)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
3. Determine when to use rational approximations and the exact values of numbers such as e, π and the irrationals.	• describing when an exact value for an irrational number may have an advantage over a rational approximation when used in problem-solving situations.

IV. Number Sense and Numeration

2. Students recognize that numbers are used in different ways like counting, measuring, ordering and estimating; understand and produce multiple representations of a number; and translate among equivalent representations. (Representation and Uses of Numbers)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
4. Apply estimation strategies in increasingly complex situations.	• using properties of exponents, irrational numbers, and logarithms in estimation strategies: - estimating products or quotients for numbers written in scientific notation, e.g., $2.9 \bullet 10^8 \bullet 8.1 \bullet 10^{15}$ is about $24 \bullet 10^{23}$ or $2.4 \bullet 10^{24}$. - estimating values of radical expressions such as $\frac{\sqrt[2]{65}}{\sqrt[3]{9}} = 8/2$ or 4.

IV. Number Sense and Numeration

2. Students recognize that numbers are used in different ways like counting, measuring, ordering and estimating; understand and produce multiple representations of a number; and translate among equivalent representations. (Representation and Uses of Numbers)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
5. Select appropriate representations for numbers, including representation of rational and irrational numbers and coordinate or vector representation of complex numbers, in order to simplify and solve problems.	• working with real-life situations that involve rational numbers, irrational numbers, exponential equations and logarithmic equations, such as: - exponential growth. - radioactive decay and carbon dating. - compound interest. - depreciation or appreciation. - the Richter scale. - decibel measure of sound.

IV. Number Sense and Numeration

3. Students investigate relationships such as equality, inequality, inverses, factors and multiples; and represent and compare very large and very small numbers. (Number Relationships)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
1. Compare and order real numbers and compare rational approximations to exact values.	• ordering two or more real numbers that are expressed in fractional, decimal, exponential, logarithmic, radical or scientific form. • choosing, discussing and defending different strategies that have been used to facilitate comparison of two numbers expressed in any forms.

IV. Number Sense and Numeration

3. Students investigate relationships such as equality, inequality, inverses, factors and multiples; and represent and compare very large and very small numbers. (Number Relationships)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
2. Express numerical comparisons as ratios and rates.	• continuing to work with numerical comparisons using ratios and rates, such as: - the basic trigonometric ratios in right triangles. - the rate of change of functions. - multiple ratios done on a spreadsheet. - ratios in corresponding lengths in similar shapes. • representing rates algebraically as well as numerically, e.g., slope of a line in the general case, $\frac{y_1 - y_2}{x_1 - x_2}$

IV. Number Sense and Numeration

3. Students investigate relationships such as equality, inequality, inverses, factors and multiples; and represent and compare very large and very small numbers. (Number Relationships)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
3. Extend the relationships of primes, factors, multiples and divisibility in an algebraic setting.	• exploring and using the concepts of relatively prime, factor, multiple, divisibility, least common multiple, and greatest common factor as they apply to algebraic expressions. Find all factors of the algebraic expression $7xy^2$. What is the greatest common factor of the expressions $x^2 + 2x + 1$ and $2x^2 - x - 3$?

IV. Number Sense and Numeration

3. Students investigate relationships such as equality, inequality, inverses, factors and multiples; and represent and compare very large and very small numbers. (Number Relationships)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
4. Express number relationships using positive and negative rational exponents, logarithms, and radicals.	• using roots to indices greater than 2. ✪ expressing radicals in exponential form, e.g., $\sqrt[3]{5^{2}=}\ 5^{\frac{2}{3}}$

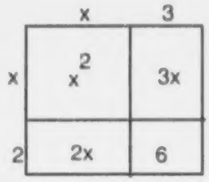
IV. Number Sense and Numeration

3. Students investigate relationships such as equality, inequality, inverses, factors and multiples; and represent and compare very large and very small numbers. (Number Relationships)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
5. Apply their understanding of number relationships in solving problems.	• solving problems that involve real-world applications where both given values and solutions include irrational, exponential, and logarithmic forms as well as rational numbers.

V. Numerical and Algebraic Operations and Analytical Thinking

1. Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems. (Operations and their Properties)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
1. Present and explain geometric and symbolic models for operations with real and complex numbers, and algebraic expressions.	• modeling real and complex numbers and their operations such as: - addition and subtraction with vectors on the number line. - a complex number as a position vector in a real/complex plane. - addition and subtraction with plane vectors using either the geometric or coordinate representation. - conjugate of a vector as the real-axis reflection of the vector ⊕ complex numbers in r(cos0 + isin0) form. ⊕ multiplication and division geometrically as rotations and size transformations (e.g., direct similarities). • modeling operations on algebraic expressions geometrically and physically as well as symbolically. $(x + 2)(x + 3)$  $x^2 + 5x + 6$

V. Numerical and Algebraic Operations and Analytical Thinking

1. Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems. (Operations and their Properties)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
2. Compute with real numbers, complex numbers, algebraic expressions, matrices and vectors using technology and, for simple instances, with paper-and-pencil algorithms.	• using paper-and-pencil algorithms for simple computations such as: - adding, subtracting and multiplying 2-by-2 matrices. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} -2 & 4 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} -4 & 14 \\ -10 & 32 \end{bmatrix}$ - multiplying/dividing real numbers represented in exponential form with the same base. - adding, subtracting, multiplying and dividing real numbers expressed with 3 digits. - adding, subtracting, multiplying and dividing simple instances of algebraic expressions. $2x \bullet (x + 3) = 2x^2 + 6x = x + 2 \qquad \frac{x^2 + 5x + 6}{x + 3} = x + 2$ - adding and subtracting complex numbers in a + bi form. - adding vectors geometrically. ⊕ operating on simple logarithmic expression such as $\log_2 5 + \log_2 3 = \log_2 15$. ⊕ multiplying and dividing complex numbers written in polar form. • using calculator or computer assistance to perform complex computations and manipulations such as: - operating on matrices with dimensions greater than 2 by 2. - performing all operations on exponential forms of real numbers. - performing all operations on real numbers expressed with any number of digits. - combining and simplifying algebraic expressions using any combination of operations. - computing with complex logarithmic expressions.

V. Numerical and Algebraic Operations and Analytical Thinking

1. Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems. (Operations and their Properties)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
3. Describe the properties of operations with numbers, algebraic expressions, vectors and matrices and make generalizations about the properties of given mathematical systems.	• investigating and describing the properties that do and do not hold for systems of numbers, algebraic expressions, matrices, and vectors, such as: - the existence of inverses. - the existence of an identity. - commutativity. - associativity. - a distributive relation between two operations. - order among elements.

V. Numerical and Algebraic Operations and Analytical Thinking

1. Students understand and use various types of operations (e.g., addition, subtraction, multiplication, division) to solve problems. (Operations and their Properties)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
4. Efficiently and accurately apply operations with real numbers, complex numbers, algebraic expressions, matrices, and vectors in solving problems.	• employing operations in situations that exhibit the usefulness of the objects being manipulated as well as being interesting to students such as: - using real number properties to develop a set of spreadsheet instructions. - using vectors to analyze linear, non-linear projectile and circular motions. - using matrices to organize and record information from business and industry. ⊕ using matrices to represent Markov Chains and adjacencies in vertex-edge graphs. • investigating situations in which the numbers involved reflect reality in that the solutions are not necessarily integral.

V. Numerical and Algebraic Operations and Analytical Thinking

2. Students analyze problems to determine an appropriate process for solution and they use algebraic notations to model or represent problems. (Algebraic and Analytic Thinking)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
1. Identify important variables in a context, symbolize them, and express their relationships algebraically.	• analyzing situations to determine important variables and relationships and representing those variables and relations algebraically as, for instance: - planning a school dance with a live dance band performance as a fund raiser. - determining a retail price to be set for a holiday tree ornament made by the combined shop and art classes. - investigating how weight and pendulum length affect the period of the pendulum. ⊕ using difference equations to help find closed form representation of a pattern.

V. Numerical and Algebraic Operations and Analytical Thinking

2. Students analyze problems to determine an appropriate process for solution and they use algebraic notations to model or represent problems. (Algebraic and Analytic Thinking)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
2. Represent algebraic concepts and relationships with matrices, spreadsheets, diagrams, graphs, tables, physical models, vectors, equations and inequalities; and translate among the various representations.	• representing relations among variables with appropriate algebraic symbolism, such as: - linear equations, inequalities or systems of linear equations. - power, exponential or circular functions. • using matrices to represent a system of linear equations. • using diagrams to show how two binomials combine when multiplied or added. • employing tables to highlight how change in one variable is affected by change in one or more other variables. • using composition of vectors to represent projectile motion and other forces.

V. Numerical and Algebraic Operations and Analytical Thinking

2. Students analyze problems to determine an appropriate process for solution and they use algebraic notations to model or represent problems. (Algebraic and Analytic Thinking)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
3. Solve linear equations and inequalities algebraically and non-linear equations using graphing, symbol-manipulating or spreadsheet technology; and solve linear and non-linear systems using appropriate methods.	• solving linear equations and inequalities algebraically and explaining the procedures used in finding the solution. • solving nonlinear equations and inequalities with a high degree of accuracy using calculator and computer-graphing, symbol- manipulating, table-making, and equation-solving capabilities. • solving algebraic systems using combinations of by-hand manipulation and computer/calculator assistance. • using the table-making, graphing/tracing, zooming and equation-solving capabilities of graphing-calculator technology to assist in solving algebraic problems.

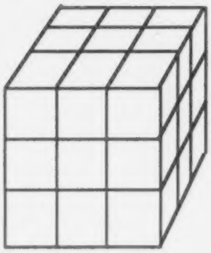
V. Numerical and Algebraic Operations and Analytical Thinking

2. Students analyze problems to determine an appropriate process for solution and they use algebraic notations to model or represent problems. (Algebraic and Analytic Thinking)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***

4. Analyze problems that can be modeled by functions, determine strategies for solving the problems, and evaluate the adequacy of the solutions in the context of the problems.

- analyzing, solving and checking solutions to problems against their contexts such as:
    - designing a bungee jump simulation that will cause no injury to any jumper.
    - determining how far a ball or other missile will travel when propelled at an acute angle to the horizontal.
    - determining the number of diagonals of a polygon with *n* sides.
    - finding the number of unit cubes with zero, one, two or three sides painted when an n-sided cube is dipped in paint and then cut into unit cubes, and defining the patterns found using functions of n.

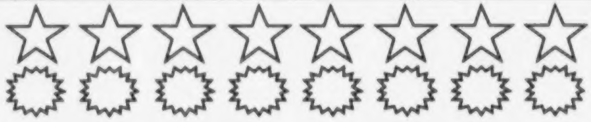


for $n = 3$
0 faces colored = 1
1 face colored = 6
2 faces colored = 12
3 faces colored = 8

# of painted sides	length of cube side					
	2	3	4	5	6	n
0	0	1	8	27	64	$f(n) = (n-2)^3$
1	0	6	24	54	96	$f(n) = 6(n-2)^2 2$
2	0	12	24	36	48	$f(n) = 12(n-2) 3$
3	8	8	8	8	8	$f(n) = 8$

V. Numerical and Algebraic Operations and Analytical Thinking

2. Students analyze problems to determine an appropriate process for solution and they use algebraic notations to model or represent problems. (Algebraic and Analytic Thinking)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
5. Explore problems that reflect the contemporary uses of mathematics in significant contexts and use the power of technology and algebraic and analytic reasoning to experience the ways mathematics is used in society.	• exploring problems that reflect contemporary uses of mathematics, such as: - matricies to encode and decode in cryptography. - vector representation of projectile motion. - geometric representation of a waiting-time distribution to determine the number of supermarket checkout or bank teller stations needed to move people through efficiently. - transformation group representation of frieze patterns and tiling patterns.  What symmetries do you see in the frieze pattern? To which of the 7 transformation groups does it belong?

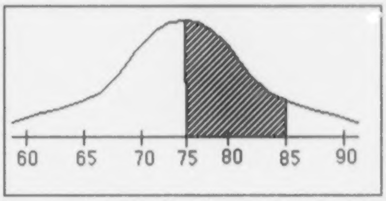
VI. Probability and Discrete Mathematics

1. Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available; and, they make critical judgments about claims that are made in probabilistic situations. (Probability)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
1. Develop an understanding of randomness and chance variation, and describe chance and certainty in the language of probability.	• exploring real-world situations where outside factors may appear to cause events, but chance variation also could be the explanation, such as: - hitting streaks in baseball or consecutive free throws made in basketball. - time needed to get from home to school on the bus. - diameters of rose blossoms when in full bloom. • describing chance situations using the language of probability.

VI. Probability and Discrete Mathematics

1. Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available; and, they make critical judgments about claims that are made in probabilistic situations. (Probability)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
2. Give a mathematical definition of probability and determine the probabilities of more complex events, and generate and interpret probability distributions.	• using the normal distribution to determine the likelihood of an event. Grades on a test are assumed to form a normal distribution with a mean score of 75 and a standard deviation of 5. Approximately what percent of students scored between 75 and 85? From 75 to 85 is two positive SDs from the mean. Two standard deviations in either direction contains ≈ 95.5% of the distribution. Therefore 95.5/2 or 47.75% would contain the positive deviations and mean that approximately 48% of the students scored in that range.  • using simulations or real data to analyze probabilistic situations, such as: - the number of draws needed to obtain a red ball from a container with reds and greens in the ratio of x/y. - the average number of similar keys that must be tried to open a lock. - the distribution of successes obtained from a population of known proportion in N trials. • graphically representing and interpreting the distribution of data generated in probabilistic situations. ✪ generating and interpreting probability distributions such as binomial, uniform, and chi square distributions.

VI. Probability and Discrete Mathematics

1. Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available; and, they make critical judgments about claims that are made in probabilistic situations. (Probability)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
3. Analyze events to determine their dependence or independence and calculate probabilities of compound events.	• determining the dependence or independence of events by analyzing events that are familiar, such as rolling dice. An experiment consists of rolling one die. Let A equal the event that 2, 3 or 4 appears. Let B equal the event that 4, 5 or 6 appears. Show that A and B are dependent events. • determining the probabilities of independent and dependent compound events, such as: - the probability of B given that A has already occurred. - the probability of A and B. - the probability of A or B. *A buyer for a gift shop will accept a shipment of 36 Swiss Clocks if a sample of 5, chosen at random, is not defective. What is the probability the buyer will accept the shipment of 36 if it contains 8 defective clocks?*

VI. Probability and Discrete Mathematics

1. Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available; and, they make critical judgments about claims that are made in probabilistic situations. (Probability)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
4. Use sampling and simulations to determine empirical probabilities and, when appropriate, compare them to the corresponding theoretical probabilities; understand and apply the law of large numbers.	• continuing to use multiple ways for generating random values for simulations as, for example: - blind draws. - a telephone book. - polyhedral dice. - random number table. - random number generator on a computer or graphics calculator. • determining empirical probabilities through sampling and simulations, comparing those probabilities to the theoretical probability when appropriate, and discussing the relative advantages of each. • investigating the meaning of the Law of Large Numbers by comparing the results of increasingly large samples with known population characteristics as, for instance: - the mean of the dates on pennies for samples of 5, 10, 20, 40, etc. from a population of 100 pennies. - the proportions of births of male and female children in hospitals in which the number of births varies from 5 to 100 per day.

VI. Probability and Discrete Mathematics

1. Students develop an understanding of the notion of certainty and of probability as a measure of the degree of likelihood that can be assigned to a given event based on the knowledge available; and, they make critical judgments about claims that are made in probabilistic situations. (Probability)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
5. Conduct probability experiments and simulations to model and solve problems, including compound events.	• solving problems using experiments and simulations such as: - exploring geometric probabilities using a Monte Carlo method.- determining the fairness of games. - exploring random behavior in genetics. - modeling a manufacturing process to determine if or when the equipment begins to produce unacceptable levels of defective products.

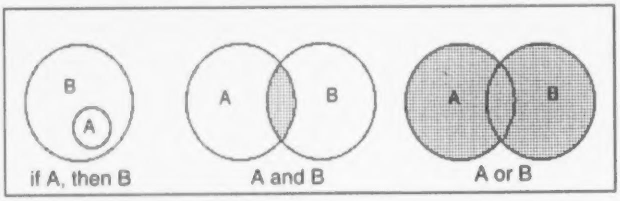
VI. Probability and Discrete Mathematics

2. Students investigate practical situations such as scheduling, routing, sequencing, networking, organizing and classifying; and analyze ideas like recurrence relations, induction and algorithm design. (Discrete Mathematics)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
1. Derive and use formulas for calculating permutations, and combinations.	• developing formulas for counting permutations and combinations from concrete examples. • using the formulas in appropriate situations with calculators facilitating evaluation. *The Jonesville soft ball team has scheduled a practice game. They have 6 infielders, 5 outfielders, 3 pitchers and 2 catchers. In how many different ways could the coach set up her starting lineup?* ${}_6C_4 \cdot {}_5C_3 \cdot {}_3C_1 \cdot {}_2C_1$

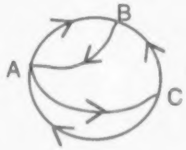
VI. Probability and Discrete Mathematics

2. Students investigate practical situations such as scheduling, routing, sequencing, networking, organizing and classifying; and analyze ideas like recurrence relations, induction and algorithm design. (Discrete Mathematics)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
2. Use sets and set relationships to represent algebraic and geometric concepts.	• analyzing and describing geometric concepts such as angle, concurrence, intersection, locus, etc. in terms of set relationships and set operations such as union and intersection. • representing algebraic concepts using sets and set relationships, such as solutions to equations, inequalities and systems of equations. • logic concepts using Venn diagrams.  B A if A, then B A B A and B A B A or B

VI. Probability and Discrete Mathematics

2. Students investigate practical situations such as scheduling, routing, sequencing, networking, organizing and classifying; and analyze ideas like recurrence relations, induction and algorithm design. (Discrete Mathematics)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
3. Use vertex-edge graphs to solve network problems such as finding circuits, critical paths, minimum spanning trees, and adjacency matrices.	• using vertex-edge graphs to solve Euler circuit problems, such as: - given a set of stops that a garbage truck must make and the roads between those stops, determine if a route can be found that will include each road exactly once and allow the truck to start from point A and return to point A. - determining whether joggers can find a path that would allow them to run on all trails in a park without retracing their steps. • using vertex-edge graphs to model situations that involve minimization as, for example: - minimizing the cost of linking a finite number of cities with telephone cable. - finding the shortest route between any two cities when given a finite set of cities and the distances between those cities that are connected by roads. • using directed graphs and adjacency matrices to determine the number of 2-stage and 3-stage indirect routes connecting any two cities in a finite situation.  $\Rightarrow \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array} = M \quad M^2 = \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$ The number of paths of length 2 = m^2

VI. Probability and Discrete Mathematics

2. Students investigate practical situations such as scheduling, routing, sequencing, networking, organizing and classifying; and analyze ideas like recurrence relations, induction and algorithm design. (Discrete Mathematics)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
4. Analyze and use discrete ideas such as induction, iteration, and recurrence relations.	• exploring, forming and testing conjectures, and presenting ideas on recursively defined situations, such as: - discrete exponential growth, e.g., in a school system, determining how many layers of calling are necessary to reach all teachers if one person calls 3 others and that pattern continues. - finding the value of an annuity after a given number of years. ⊕ changing recursively defined relations to closed form using difference equations. • analyzing the patterns generated by iteration: - those found in fractals. - iteration of functions using a calculator. ⊕ graphical analysis of function iteration such as in dynamical systems. ⊕ proof by mathematical induction.

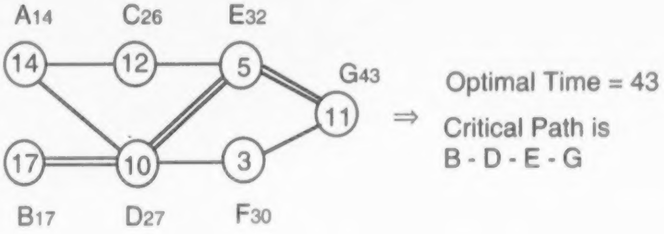
VI. Probability and Discrete Mathematics

2. Students investigate practical situations such as scheduling, routing, sequencing, networking, organizing and classifying; and analyze ideas like recurrence relations, induction and algorithm design. (Discrete Mathematics)

In the secondary years, students have the opportunity to...	*By engaging in activities such as...*
5. Design and analyze efficient algorithms to accomplish a task or solve a problem in a variety of contexts including practical, mathematical, and computer-related situations.	• designing algorithms to accomplish a variety of tasks: - playing games such as tic-tac-toe or nim. - constructing a geometric figure. - developing a flow diagram for a calculator or computer program. - determining a method to find a critical path through a network. - employing the nesting method for evaluating a polynomial on a calculator.

VI. Probability and Discrete Mathematics

2. Students investigate practical situations such as scheduling, routing, sequencing, networking, organizing and classifying; and analyze ideas like recurrence relations, induction and algorithm design. (Discrete Mathematics)

In the secondary years, students have the opportunity to...	***By engaging in activities such as...***
6. Use discrete mathematics concepts as described above to model situations and solve problems; look for whether or not there is a solution (existence problems), determine how many solutions there are (counting problems), and decide upon a best solution (optimization problems).	• using discrete mathematics concepts to model situations and solve problems, such as: - bin packing: given a set of items of various sizes and bins of a fixed capacity, determine the smallest number of bins that are necessary to hold all of the items. - PERT Charts: given a set of tasks requiring different lengths of time to complete, time and order constraints on the tasks, and a set number of processors, determine how the tasks should be sequenced to take the shortest amount of time.  A14 C26 E32 14 12 5 G43 11 17 10 3 B17 D27 F30 ⇒ Optimal Time = 43 Critical Path is B - D - E - G - using linear programming to find the maximum profit in a situation or to find the minimum cost to manufacture a product given appropriate constraints. - employing graph-coloring methods to minimize the potential conflicts in scheduling meetings for a number of committees with joint membership.

Additional Resource Publications

Guide to Selecting Instructional Materials for Mathematics Education

The selection of appropriate instructional materials is critical to the implementation of effective mathematics instruction. Available materials are in a state of evolution influenced largely by expanding technology. Educators are no longer limited to the use of textbooks alone. To keep abreast of the changing paradigm for mathematics instruction, the Association of State Supervisors of Mathematics (ASSM) and the National Council of Supervisors of Mathematics (NCSM) established a joint Instructional Materials Committee to prepare this set of guidelines for the selection of instructional materials in mathematics.

Mathematics Teaching NCATE/NCTM (September 1993)

Grades: K-4 5-8 7-12

Teaching Preparation programs.

High School Proficiency Test in Mathematics

The Assessment Framework for the Michigan High School Proficiency Test in Mathematics was developed by the Michigan Council of Teachers of Mathematics (MCTM) under contract with the Michigan Department of Education.

The Framework has been developed by Michigan classroom teachers, curriculum coordinators school administrators, teacher educators and business representatives. A broad representation of Michigan's diverse population has been involved with the project.

Early Childhood Standards of Quality

PreKindergarten Through Second Grade
Learner Outcomes that Promote Children's Mathematical Development

Children need to develop mathematical foundations that are broader in scope than numbers and counting. These foundations enable them to use and expand their knowledge to meet the mathematical demands they will face in the future.

Physical and mental interactions with the environment, materials, and other individuals give children opportunities to construct, modify, and integrate mathematical concepts. As they move through these learner outcomes, children will increase their understanding and skill in mathematical development.

World Wide Web Resource Links

(Requires and active connection to the Internet and a properly installed browser.)

Organizations

NCTM (National Council of Teachers of Mathematics)
http://www.nctm.org/

MCTM (Michigan Council of Teachers of Mathematics)
http://mictm.org/

NSTA (National Science Teachers Association)
http://www.nsta.org/

MDE (Michigan Department of Education)
http://www.mde.state.mi.us/

Reference

Merriam Webster Dictionary
http://www.m-w.com/dictionary

Wordsmyth English Dictionary-Thesaurus
http://www.lightlink.com/bobp/wedt/

Virtual Reference Desk
http://www.refdesk.com/dictsrch.html

World Fact Book
http://www.odci.gov/cia/publications/factbook/index.html

Acronym and Abbreviations
http://www.ucc.ie/info/net/acronyms/index.html

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